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Application of the Global Positioning System to Crustal Deformation Measurement
2. The Influence of Errors in Orbit Determination Networks

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Global Positioning System (GPS) measurements of a geodetic network in southern and central California have been used to investigate the errors introduced by adopting different sets of stations as fixed. Such fixed points, called fiducial stations, are necessary to eliminate the errors of imprecise satellites orbits, which otherwise would dominate the error budget for distances greater than tens of kilometers. These fiducial stations also define the reference frame of the crustal deformation network. Establishing the magnitude of the effect of changing the fiducial network is essential for crustal deformation studies, so that these artifacts of the differences between fiducial networks used for the data analyses are not interpreted as geophysical signals. Solutions for a crustal deformation network spanning distances up to 350 km were computed with a variety of fiducial networks. We use fiducial coordinates determined from very long baseline interferometry (VLBI). We compare these solutions by computing the equivalent uniform strain and rotation that best maps one solution into another. If we use a continental-scale fiducial network with good geometry, the distortions between the solutions are about 10^{-8}, largely independent of the exact choice of stations. The one case of a large- scale fiducial network where the distortions are larger is when the three fiducial stations chosen all lie close to a great circle. Use of a fiducial network no larger than the crustal deformation network can produce apparent strains of up to 10^{-7}. Our work suggests that fiducial coordinates determined from GPS data analysis may be used, although they should be determined using a consistent reference frame, such as provided by VLBI and satellite laser ranging.

INTRODUCTION

The Global Positioning System (GPS) is revolutionizing the way in which high-accuracy crustal deformation experiments are being conducted. Previously, small-scale information on crustal deformation came from repeated electronic distance measurements (EDM), with precisions of 3 mm + 1 part in 10^7. This system measures only the length between stations with intervisibility and is limited to distances of approximately 50 km [*Savage and Prescott*, 1973]. On longer scales, over distances of thousands of kilometers, very long baseline interferometry (VLBI) and satellite laser ranging (SLR) have been used to measure contemporary relative plate motions [*Herring et al.*, 1986; *Clark et al.*, 1987; *Smith et al.*, 1990]. The strength of GPS over VLBI, SLR, and EDM is its ability to deliver three-dimensional vector positions easily and inexpensively between sites that are not intervisible.

Recent analyses of GPS measurements collected over a few days show that the scatter in the horizontal components of interstation vector estimates is a few millimeters for baselines out to distances of a few hundred kilometers, with a proportional error of perhaps 10^{-8} becoming apparent at greater distances [*Dong and Bock*, 1989; *Blewitt*, 1989; *Larson*, 1990a]. In these works, fiducial networks were used to compute precise satellite orbits. A fiducial network consists of ground stations which observe satellites that are also being observed by stations in the crustal deformation experiment. When the three-dimensional locations of these "fiducial" sites are known to better than several centimeters, the orbit of the GPS satellite can be estimated with great accuracy. In turn, improved knowledge of the satellite orbit improves the precision and accuracy with which interstation vectors within the crustal deformation network are known, relative to an Earth-fixed reference frame.

This is the second in a series of papers devoted to characterizing errors in GPS data and GPS modeling techniques. The first [*Larson and Agnew*, this issue] (hereinafter referred to as paper 1) calculated precision and accuracy from nearly 3 years of surveys in southern and central California. In order to provide a large sample of vector measurements, that work included analysis of data with a large variety of fiducial networks. The decision to use these fiducial networks was dictated by the scarcity of high-quality fiducial data. We wish to determine how much scatter in the data described in paper 1 is attributable to the a priori choice of fiducial network.

The choice of fiducial network is important for two reasons. First, fiducial networks affect the precision that can be achieved with a single GPS experiment, which in this

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period was generally conducted over 7 to 8 hours. Future crustal deformation experiments can benefit from this study, because it demonstrates that precision and accuracy can be optimized by choosing the correct fiducial stations. Secondy, crustal deformation studies, by definition, require the remeasurement of interstation vectors during several different experiments, usually over a several year period. Since there is no guarantee that each experiment will have useful data from the same network of fiducial stations, it is necessary to address the bias which is introduced by the choice of fiducial network. Specifically, if one is forced to use these different fiducial networks at different epochs over the course of the crustal deformation experiment, we want to be able to choose the fiducial networks for each experiment such that the fiducial network bias is minimized. Since fiducial networks define the scale and orientation of the reference frame, we expect that the effect of the fiducial network bias on the crustal deformation network would be equivalent to a network dilatation and rotation. That is, if the fiducial networks from each experiment are poorly matched, the bias of each fiducial network on the crustal deformation network may look like crustal strain. These effects may not be observed in the scatter, or short- term precision, calculated from several days of data. Establishing how the choice of fiducial network affects the interstation vector estimates within the crustal deformation network is essential, so that these biases are not interpreted as geophysical signals. We first present an overview of fiducial networks, how they work, and how we expect them to be affected by different sources of error. Subsequently, we describe our procedure for quantifying the effect of different fiducial networks on interstation vector estimates. Finally, we present results from our analysis of data and discuss how biases in fiducial networks can be taken into account in the final error analysis.

FIDUCIAL NETWORKS

The optimal size of a fiducial network ideally depends on the size of the crustal deformation network. However, the requirement of common satellite visibility between the fiducial stations and the crustal deformation stations places a practical limitation on the size and shape of the fiducial network. Generally, fiducial networks should be an order of magnitude larger than the geodetic network. This is because errors in the satellite's orbit are proportional to the scale of the fiducial network. These satellite orbit errors map proportionally into the baseline errors in the geodetic network. For the purposes of measuring crustal deformation in California [paper 1], where networks ranged in scale from 50 to 450 km, fiducial networks which span continental distances (2000-3000 km) are required. At these length scales, VLBI and Satellite Laser Ranging (SLR) provide the only sources of precise fiducial coordinates. Because the coordinates of the fiducial sites are fixed, the known measurement uncertainties in these coordinates are ignored. These uncertainties in fiducial coordinates, which are a combination of the uncertainty in the VLBI or SLR measurement history and errors in the local survey from the VLBI or SLR monument to the GPS monument, depend on which fiducial sites are chosen and the quality of the local surveys. Later in this paper we will address the issue of incorporating uncertainties of fiducial coordinates into the error budget.

Simple geometrical arguments can be used to describe the effect of fiducial networks on GPS baseline estimation. Letting f be the distance between two stations in a fiducial network and $|df|$ be the error in that baseline, then the upper bound on the magnitude of the error in a satellite's orbit, $|dr|$, is approximated by

$$\frac{|dr|}{r} \cong \frac{|df|}{f} \tag{1}$$

where r is the range to the satellite. Similarly, this error in the satellite orbit would be equal to the error, $|dx|$, on a baseline of length x in the geodetic network,

$$\frac{|dx|}{x} \cong \frac{|dr|}{r} \cong \frac{|df|}{f} \tag{2}$$

For example, if the fiducial network spans 3000 km on each side, equation (2) suggests that an error of 30 mm in the fiducial network coordinates will cause a 1 part in 10^8 error. The effect of the same absolute error on a fiducial network which spans 300 km on each side is an order of magnitude larger, 1 part in 10^7. On a 10-km baseline, a part in 10^7 yields an acceptable mm level error but results in a 10-mm error on a 100-km baseline. So, if one is using GPS to monitor short baselines, less than 10 km, the stations chosen for the fiducial network may not be important. At the simplest level, a fiducial network can be eliminated entirely by using broadcast ephemerides. This orbit information is determined by the Defense of Department (DOD) which operates the Global Positioning System. The precision and accuracy of these orbits can be intentionally degraded by the DOD at any time. Use of broadcast ephemerides places the interstation vector estimates in the reference frame of the DOD's tracking network. High-accuracy results at distances of 10-30 km using broadcast orbits have been described by *Prescott et al.* [1989]. But for measurements outside this spatial regime, the choice of fiducial network may significantly contribute to the fiducial bias of interstation vectors in the crustal deformation network.

The effect of fiducial networks on system precision and accuracy was studied on continental-scale baselines in North America by *Lichten et al.* [1989] with data collected during the November 1985 North American GPS experiment. Unlike a covariance analysis, studies using data take into account error sources associated with data gaps and ambiguity resolution. Lichten et al. used combinations of data from four GPS sites collocated with VLBI monuments (Westford, Massachusetts; Richmond, Florida; Fort Davis, Texas; and Hatcreek, California), fixing the position of three and estimating the position of the fourth. They then examined the differences between the interstation vector between the estimated site and the closest fiducial site and the VLBI-derived interstation vector. They found that the effect of changing the fiducial network (i.e., changing one site from fixed to estimated) was of the order of 1 part in 10^8 of the baseline length. The effect of altering fiducial networks on shorter, regional-scale interstation vectors has not been investigated elsewhere.

Until recently, there has been little opportunity to study the effect of changing the fiducial network on estimates of interstation vectors, because of the scarcity of more than three fiducial stations in any one experiment. An important objective of this paper is to investigate whether all fiducial networks determined from VLBI have the same effect on estimates of interstation vectors, particularly if the geometry of fiducial sites is a contributing factor to the precision and accuracy of the fiducial network.

HOW TO COMPUTE FIDUCIAL COORDINATES

We derived our fiducial coordinates from a global VLBI velocity model, GLB223, provided by Goddard Space Flight Center (C. Ma, personal communication, 1988). In this solution, a least squares fit was made of the VLBI data, and initial positions and velocities of the VLBI sites, Earth orientation, and nutation were solved for. The azimuth from Westford to Fairbanks (Gilcreek) was held fixed. Westford's velocity was defined by AM-02 [*Minster and Jordan,* 1978], and the reference epoch of the initial positions was October

TABLE 1. VLBI Model GLB223

Site	CDP		Adjusted Value	Unscaled Error
Westford	X Component	7209	1492208554.00 mm	reference
Westford	Y Component	NOAM	-4458131329.00 mm	reference
Westford	Z Component		4296015877.00 mm	reference
Westford	X Velocity		-18.23 mm/yr	reference
Westford	Y Velocity		-2.85 mm/yr	reference
Westford	Z Velocity		3.38 mm/yr	reference
Algonquin	X Component	7282	918036730.70 mm	2.538 mm
Algonquin	Y Component	NOAM	-4346133033.33 mm	8.788 mm
Algonquin	Z Component		4561971539.22 mm	8.623 mm
Algonquin	X Velocity		-19.26 mm/yr	0.00 mm/yr
Algonquin	Y Velocity		-2.86 mm/yr	0.00 mm/yr
Algonquin	Z Velocity		1.16 mm/yr	0.00 mm/yr
Fort Ord	X Component	7266	-2697024787.76 mm	7.63 mm
Fort Ord	Y Component		-4354394350.65 mm	14.32 mm
Fort Ord	Z Component		3788078018.22 mm	15.92 mm
Fort Ord	X Velocity		-26.85 mm/yr	1.00 mm/yr
Fort Ord	Y Velocity		34.22 mm/yr	1.15 mm/yr
Fort Ord	Z Velocity		20.24 mm/yr	1.32 mm/yr
Mojave	X Component	7222	-2356169113.12 mm	2.97 mm
Mojave	Y Component	NOAM	-4646756751.66 mm	9.73 mm
Mojave	Z Component		3668471206.34 mm	11.96 mm
Mojave	X Velocity		-16.42 mm/yr	0.45 mm/yr
Mojave	Y Velocity		5.83 mm/yr	0.65 mm/yr
Mojave	Z Velocity		-3.22 mm/yr	0.72 mm/yr
Ovro	X Component	7207	-2409598847.92 mm	2.64 mm
Ovro	Y Component	NOAM	-4478350412.58 mm	9.55 mm
Ovro	Z Component		3838603810.08 mm	11.98 mm
Ovro	X Velocity		-18.61 mm/yr	0.41 mm/yr
Ovro	Y Velocity		7.15 mm/yr	0.68 mm/yr
Ovro	Z Velocity		-3.40 mm/yr	0.73 mm/yr
Platteville	X Component	7258	-1240706258.17 mm	7.51 mm
Platteville	Y Component	NOAM	-4720455192.09 mm	12.01 mm
Platteville	Z Component		4094482213.75 mm	13.3 mm
Platteville	X Velocity		-15.00 mm/yr	1.28 mm/yr
Platteville	Y Velocity		2.40 mm/yr	1.32 mm/yr
Platteville	Z Velocity		-1.81 mm/yr	1.52 mm/yr
Palos Verdes	X Component	7268	-2525450843.30 mm	15.10 mm
Palos Verdes	Y Component	PCFC	-4670036683.10 mm	21.20 mm
Palos Verdes	Z Component		3522887273.70 mm	20.40 mm
Palos Verdes	X Velocity		-28.30 mm/yr	1.70 mm/yr
Palos Verdes	Y Velocity		24.30 mm/yr	1.40 mm/yr
Palos Verdes	Z Velocity		12.00 mm/yr	1.90 mm/yr
Richmond	X Component	7219	961259853.99 mm	4.08 mm
Richmond	Y Component	NOAM	-5674090905.67 mm	6.24 mm
Richmond	Z Component		2740534229.84 mm	5.51 mm
Richmond	X Velocity		-14.40 mm/yr	0.71 mm/yr
Richmond	Y Velocity		-2.20 mm/yr	0.32 mm/yr
Richmond	Z Velocity		0.51 mm/yr	0.66 mm/yr
Vandenberg	X Component	7223	-2678092760.68 mm	4.00 mm
Vandenberg	Y Component	PCFC	-4525451894.49 mm	10.90 mm
Vandenberg	Z Component		3597410581.61 mm	13.20 mm
Vandenberg	X Velocity		-31.77 mm/yr	0.56 mm/yr
Vandenberg	Y Velocity		34.26 mm/yr	0.74 mm/yr
Vandenberg	Z Velocity		19.48 mm/yr	0.82 mm/yr

GLB223 global solution for fiducial stations used in paper 1 and this study. Westford's velocity is defined by AM0-2. The azimuth from Westford to Fairbanks is held fixed. The epoch of the initial positions is October 17, 1980. Solution provided by Goddard Space Flight Center (C. Ma, personal communication, 1988). CDP numbers are defined by *Noll* [1988]. PCFC stands for Pacific plate, NOAM for North American plate.

17, 1980. The initial position vectors $\mathbf{r}_{initial}$ and velocity vectors $\mathbf{v}$ are given in Table 1. For any epoch t, the coordinates of the VLBI monument are

$$\mathbf{r}_{vlbi}(t) = \mathbf{r}_{intial} + (t - t_{ref}) \cdot \mathbf{v} \qquad (3)$$

VLBI is not sensitive to the geocenter, so we made a correction to all stations, using values derived from SLR measurements (M. Murray, personal communication, 1990). GPS fiducial coordinates were then calculated using the following equation:

$$\mathbf{r}_{gps}(t) = \mathbf{r}_{vlbi}(t) + \mathbf{r}_{gc} + \sum \mathbf{r}_{ls} \qquad (4)$$

where $\sum \mathbf{r}_{ls}$ is the vector sum of all local survey vectors between GPS and VLBI reference monuments and $\mathbf{r}_{gc}$ is the offset of the VLBI reference frame from the geocenter. More recent Goddard Space Flight Center VLBI models now incorporate the geocenter offset (J. Ray, personal communication, 1991).

Data Analysis

These data were collected during the March 1988 central California experiment, which is described in paper 1 (there referred to as M88b). We define two kinds of sites: crustal deformation sites and fiducial sites. Fiducial sites are those which have been measured by VLBI and whose positions can be computed by equation (4). The positions of crustal deformation sites were estimated. Fiducial sites are shown in Figure 1. Crustal deformation sites (10 in all) are shown in paper 1. The interstation vectors which make up the crustal deformation network are listed in Table 4 of paper 1. Baselines range from 50 to 350 km in length. Although data retention in the crustal deformation network was nearly 100%, the fiducial sites were less reliable, with most losing at least 1 of 4 days available. The amount of data available at each fiducial site is listed in Table 2, in numbers of hours of station-satellite tracking. Thus, 4 hours viewing of seven satellites would yield 28 hours. The models we have used

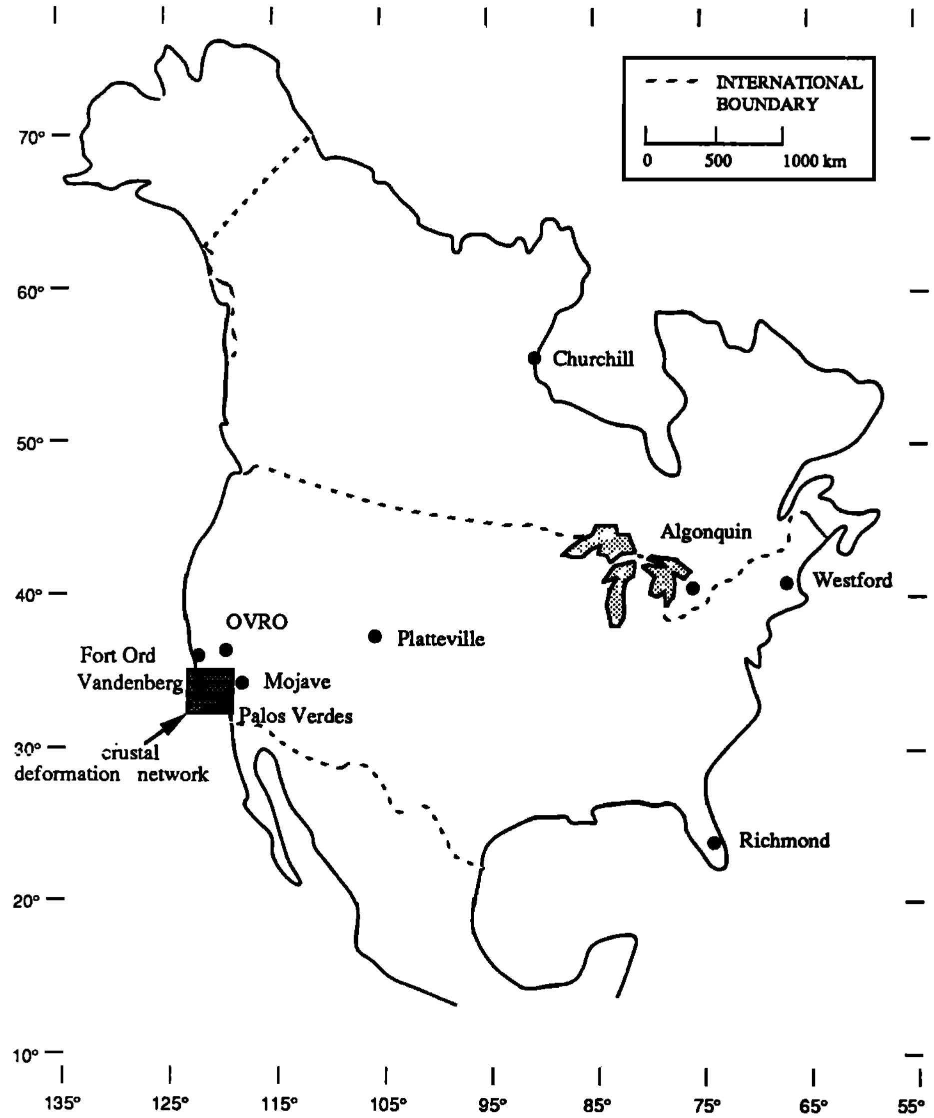

Fig. 1. North American fiducial sites. Westford, Richmond, and Mojave were continuously operating CIGNET sites [*Chin*, 1988]. Fort Ord, OVRO, Platteville, Algonquin, and Churchill were tracked with portable receivers. Coordinates of all sites (except for Churchill) were determined from VLBI measurements. Coordinates of Churchill were determined from GPS data discussed in this paper.

TABLE 2. Amount of Data Available at Each Fiducial Site

	Day of Year											
	75			76			77			78		
Station	Data, hours	ϕ_r, mm	P_r, mm	Data, hours	ϕ_r, mm	P_r, mm	Data, hours	ϕ_r, mm	P_r, mm	Data, hours	ϕ_r, mm	P_r, mm
Algonquin	27.7	3.9	900	27.6	4.2	993	27.2	4.3	1038	27.2	4.4	968
Churchill	23.7	4.0	836	16.4	-	-	24.1	4.1	878	25.0	4.3	889
Fort Ord	22.8	4.8	767	24.7	4.4	865	24.6	5.0	854	24.5	5.6	886
Mojave	0	-	-	20.5	7.3	1179	20.1	8.2	1267	28.4	7.5	1296
OVRO	21.8	5.2	829	25.5	4.9	812	13.7	-	-	19.2	5.0	777
Palos Verdes	24.8	5.5	1158	25.0	5.4	1247	24.6	5.8	1159	24.8	5.5	1127
Platteville	30.2	4.0	740	0	-	-	30.1	4.2	741	30.3	4.1	968
Richmond	27.8	5.1	1247	28.6	4.9	1048	10.1	-	-	28.8	5.7	1190
Vandenberg	0	-	-	25.0	3.6	718	21.5	3.7	763	24.5	3.5	783
Westford	26.6	4.6	1308	26.0	4.6	1368	26.8	4.6	1357	26.4	5.8	1330

ϕ_r carrier phase residual; P_r pseudorange residual.

to analyze these data were described in paper 1. We also list in Table 2 the computed postfit residual RMS for each fiducial station for each day of the experiment. We will use this information to support our arguments that certain combinations of fiducial sites produce less accurate orbits than others and that the amount and quality of data are not a contributing factor in this data set.

We test the effects of fiducial networks in a simple manner. We estimate all parameters identically and change which three fiducial sites we fix. We then examine the differences in the estimated coordinates of the crustal deformation network. We would like to be able to determine the precision and accuracy of the crustal deformation measurements. For precision, we compute the weighted RMS about the weighted mean of the daily estimates. Assessing the accuracy of different fiducial techniques from a single data set is difficult. So-called "single epoch" comparisons with VLBI are inadequate if there are survey errors between the GPS monument and VLBI reference monument (paper 1). VLBI comparisons also limit the scope of the study to a few baselines. Rather than accuracy, we will instead determine the local network stability given a particular fiducial configuration.

The relative effect of two different fiducial networks is quantified by estimating the effective mean local strain on the crustal deformation network resulting from the different fiducial networks used. The geodetic coordinates (longitude λ', latitude ϕ', and elevation h') of the ith crustal deformation site calculated with one fiducial network can be described with respect to the crustal deformation coordinates calculated using a reference fiducial network (λ, ϕ, h) in the following manner

$$\mathbf{u}_i' = \mathbf{u}_i + \mathbf{C}(\mathbf{u}_i - \mathbf{u}_r) \tag{5}$$

where $\mathbf{u}_i'$ and $\mathbf{u}_i$ are the vectors of the geodetic coordinates calculated using the two fiducial networks and $\mathbf{C}$ is the distortion in the station coordinates caused by the difference in fiducial networks. The subscript r refers to the coordinates of a reference station, which is defined here as Vandenberg. Explicitly, equation (5) has the form

$$\begin{vmatrix} \lambda' \\ \phi' \\ h' \end{vmatrix} = \begin{vmatrix} \lambda \\ \phi \\ h \end{vmatrix} + \begin{vmatrix} C_{\lambda\lambda} & C_{\lambda\phi} & 0 \\ C_{\phi\lambda} & C_{\phi\phi} & 0 \\ C_{h\lambda} & C_{h\phi} & 0 \end{vmatrix} \begin{vmatrix} \lambda - \lambda_r \\ \phi - \phi_r \\ h - h_r \end{vmatrix} \tag{6}$$

and follows the formulation of *Drew and Snay* [1989] for the analysis of crustal strain rates from geodetic data. Note that the elevation differences between the two sets of coordinates are strictly a function of the horizontal position of the station. That is, there is no vertical distortion, only horizontal distortions (strains) and network tilting.

$\mathbf{C}$ can then be decomposed into a horizontal displacement matrix, $\mathbf{L}$,

$$\mathbf{L} = \begin{vmatrix} \epsilon_{EE} & \epsilon_{EN} \\ \epsilon_{NE} & \epsilon_{NN} \end{vmatrix} = \begin{vmatrix} C_{\lambda\lambda} & \frac{C_{\lambda\phi}+R_p}{R_m} \\ \frac{C_{\phi\lambda}+R_m}{R_p} & C_{\phi\phi} \end{vmatrix} \tag{7}$$

and a network tilt vector $\mathbf{T}$:

$$\mathbf{T} = \begin{vmatrix} \tau_E \\ \tau_N \end{vmatrix} = \begin{vmatrix} \frac{-C_{h\lambda}}{R_p} \\ \frac{-C_{h\phi}}{R_m} \end{vmatrix} \tag{8}$$

In this representation, $\mathbf{L}$ and $\mathbf{T}$ refer to a planar coordinate system at the reference station, where the constants R_p and

R_m are given by

$$R_p = \frac{a}{W} cos\phi_r \quad (9)$$

$$R_m = \frac{a}{W^3}(1 - e^2) \quad (10)$$

$$W^2 = 1 - e^2 sin^2\phi_r \quad (11)$$

The constants e and a are the eccentricity and length of the semimajor axis of the WGS84 reference ellipsoid, respectively.

In terms of evaluating the differences between fiducial networks, dilatation, rotation, and maximum tilt are useful parameters to compute from the horizontal and vertical strain components. The areal dilatation Δ is the sum of the strains in the east and north components:

$$\Delta = \epsilon_{EE} + \epsilon_{NN} \quad (12)$$

The clockwise rotation of the network is defined

$$\omega = \frac{\epsilon_{EN} - \epsilon_{NE}}{2} \quad (13)$$

Finally, the maximum tilt τ_{max} is defined

$$\tau_{max} = \sqrt{\tau_E^2 + \tau_N^2} \quad (14)$$

The areal dilatation Δ describes the expansion or contraction of the crustal deformation network, and the maximum tilt τ_{max} describes the vertical differences in the networks. Rotation ω is the rigid body motion of the crustal deformation network. For completeness, we also calculate the engineering strains γ_1 and γ_2:

$$\gamma_1 = \epsilon_{EE} - \epsilon_{NN} \quad (15)$$

$$\gamma_2 = \epsilon_{EN} + \epsilon_{NE} \quad (16)$$

To summarize network precision, we first calculate the precision of each baseline vector in the crustal deformation network, using the weighted RMS about the weighted mean, as defined in paper 1. We then use a simple linear relation to describe the variation of precision σ with baseline length l:

$$\sigma = A + Bl \quad (17)$$

Network precision and the results of our strain analysis are listed in Tables 3 and 4.

RESULTS

We first examine fiducial networks which span the North American continent, and are geometrically strong (i.e., networks that approximate an equal angle triangle). The interstation distances range from 3000 to 4000 km. Examples of geometrically robust fiducial networks can be seen in Figure 1, where Westford and Richmond are combined with a VLBI site in California, either Mojave, OVRO, or Fort Ord. Although this configuration produces orbits which are valid over the entire continent, the crustal deformation network, shown in grey, is only a small portion of the larger network. Vandenberg conceivably could be used with Westford and Richmond as well. Because we are interested in the motion of Vandenberg, we chose to estimate its position. In these three configurations, errors at Westford and Richmond would contribute equally, but errors at the three California sites are different. The Fort Ord GPS occupations have been at the VLBI monument. OVRO and Mojave have been more frequently measured with VLBI than Fort Ord (thus their coordinates are known with higher accuracy), but one must use local surveys to determine the coordinates of the fiducial sites at OVRO and Mojave. The OVRO GPS site is a VLBI reference mark, whereas the GPS site at Mojave is tied through a local ground survey to a VLBI reference mark, adding to the uncertainty in the station coordinates. Even so, the agreement of GPS and VLBI determined in-

TABLE 3. Interstation Vector Precision

Fiducial Network	East		North		Vertical		Length	
	A, mm	B, 10^8	A, mm	B, 10^8	A, mm	B, 10^8	A, mm	B, 10^8
Westford-Platteville-Fort Ord	1.6	1.4	0.9	1.7	12	11.3	1.1	1.1
Westford-Richmond-Fort Ord	2.1	1.3	1.9	0.6	17	0.1	1.5	1.0
Westford-Richmond-OVRO	1.6	1.8	1.7	0.2	17	0.1	1.6	1.0
Algonquin-Richmond-Fort Ord	3.0	0.4	1.8	0.2	17	−1.2	2.2	0.5
Westford-Richmond-Mojave	3.2	0.2	1.2	0.4	13	1.8	2.5	0.5
Mojave-OVRO-Palos Verdes Vandenberg-Fort Ord (20 mm)	3.8	1.4	1.0	0.3	21	2.3	2.8	1.1
Mojave-OVRO-Palos Verdes Vandenberg-Fort Ord (40 mm)	1.6	5.9	0.9	0.7	19	5.2	0.4	5.5

We compute the weighted RMS about the mean for the interstation vectors in the central California crustal deformation network. These short-term precision estimates σ are then summarized by a fit to $\sigma = A + B \cdot length$, as discussed in paper 1. The network consisting of Mojave-OVRO-Palos Verdes-Vandenberg-Fort Ord is shown for two cases, where fiducial sites were estimated with standard deviations of 20 or 40 mm. In both cases, Mojave was fixed.

TABLE 4. Effective Strain Components

	Dilatation Δ, 10^{-8} rad	Rotation ω, 10^{-8} rad	Maximum Tilt, 10^{-8} rad	γ_1, 10^{-8} rad	γ_2, 10^{-8} rad
Continental scale/good geometry 1, Westford-Richmond-Fort Ord 2, Westford-Richmond-OVRO	0.83	-0.15	2.20	0.72	0.15
Continental scale/one common station 1, Algonquin-Richmond-Fort Ord 2, Westford-Richmond-OVRO	0.68	-0.83	3.97	1.89	-1.06
Continental scale/poor geometry 1, Westford-Richmond-Fort Ord 2, Westford-Platteville-Fort Ord	7.87	-1.63	21.35	-2.46	-3.63
Local scale/five VLBI sites 1, Westford-Richmond-Fort Ord 2, Mojave-OVRO-Palos Verdes-Vandenberg-Fort Ord[a]	-8.36	-3.24	5.36	4.99	-1.84
Local scale/three VLBI sites 1, Westford-Richmond-Fort Ord 2, Mojave-Palos Verdes-Vandenberg[a]	-3.51	-5.25	18.10	22.46	-3.37

[a]Mojave fixed, other sites estimated with a standard deviation of 20 mm.

terstation vectors between OVRO, Fort Ord, and Mojave in paper 1 implies that the survey errors are no larger than 20 mm at each site.

The short-term precision for networks consisting of Westford, Richmond, and one of the three California VLBI sites, as listed in Table 3, are similar to each other, several millimeters plus 1 part in 10^8 of the baseline length for horizontal components. The dilatation between Westford-Richmond-OVRO and Westford-Richmond-Fort Ord is 0.8 parts in 10^{-8}, and the rotation is -0.15×10^{-8} rad. The maximum tilt between Westford-Richmond-OVRO and Westford-Richmond-Fort Ord is 2.2×10^{-8} rad. These statistics imply that the effect of substituting one fiducial network for another is at the 1 part in 10^8 level. Although the computed postfit residual RMS for Mojave is significantly higher than either Fort Ord or OVRO (Table 2), which may reflect site conditions at Mojave, it does not appear to affect the estimates of interstation vectors in the crustal deformation network, as indicated by the strain calculation between Westford-Richmond-Fort Ord and Westford-Richmond-Mojave.

Westford was frequently used as a fiducial site for GPS experiments conducted in North America between 1986 and 1988. It has well-determined VLBI coordinates and easily available data, since it is part of the continuously operating tracking network, CIGNET [*Chin*, 1988]. It might be necessary or desirable to occupy an alternative site. Algonquin Park, in Ontario, Canada, is geographically similar to Westford and has been measured with VLBI at two epochs, once in 1984 and again in 1985. A VLBI reference mark at Algonquin was occupied in March 1988, thus allowing us to compare solutions with and without Westford. Algonquin has been used as a fiducial site by others [*Dong and Bock*, 1989]. We have examined the network solutions for Algonquin-Richmond-Fort Ord and find that the short-term precision for horizontal components is comparable to that of Westford-Richmond-OVRO and Westford-Richmond-Fort Ord. The internal agreement between Alqonquin-Richmond-Mojave, Algonquin-Richmond-OVRO, and Algonquin-Richmond-Fort Ord is likewise similar to that when Westford was used instead of Algonquin.

The best test of whether the fiducial coordinates are internally consistent would be to compare fiducial networks which have no stations in common. There is not a sufficient distribution of VLBI stations in this data set, so instead we have compared fiducial networks Westford-Richmond-OVRO and Algonquin-Richmond-Fort Ord. The common stations in the networks have been reduced to one: Richmond. Both scale and rotation will then be independent, except of course for the common source of coordinates: VLBI. Figure 2 shows the length and north-south, east-west, and vertical component solutions, as a function of baseline length, for interstation vectors in the crustal deformation network. The Westford-Richmond-OVRO mean solution is plotted relative to the Algonquin-Richmond-Fort Ord mean solution. The agreement is within 2 mm for both horizontal components. The maximum vertical difference is 10 mm. The areal dilatation of the two networks is 0.7×10^{-8}, and the rotation is -0.8×10^{-8} rad. The maximum tilt is comparable to those when both Westford and Richmond were fiducial sites, and different sites in California were used, 3.9×10^{-8} rad. In addition to the evidence that different fiducial sites can be used without serious deterioration of precision, this result also confirms that the VLBI derived site velocity model, GLB223, is internally consistent and that local survey errors between VLBI and GPS monuments do not significantly degrade fiducial stability.

Having established that fixing three collocated VLBI sites is adequate for precise orbit determination and creates a stable reference frame, another aspect of "fiducial data quality" that needs to be considered is the geometry of the fiducial stations. Optimal fiducial networks were not always available during actual GPS field experiments conducted in California between 1986 and 1989 (see paper 1 for further details). Rather than discard measurements made with a poor fiducial network, it is important to at least have a qualitative estimate of the bias induced by using a suboptimal geometric network. Figure 1 includes three VLBI sites which suffer from "poor" network geometry: Westford, Platteville, and Fort Ord. By poor geometry, we mean stations that are nearly colinear. *Dong and Bock* [1989] used a similar fidu-

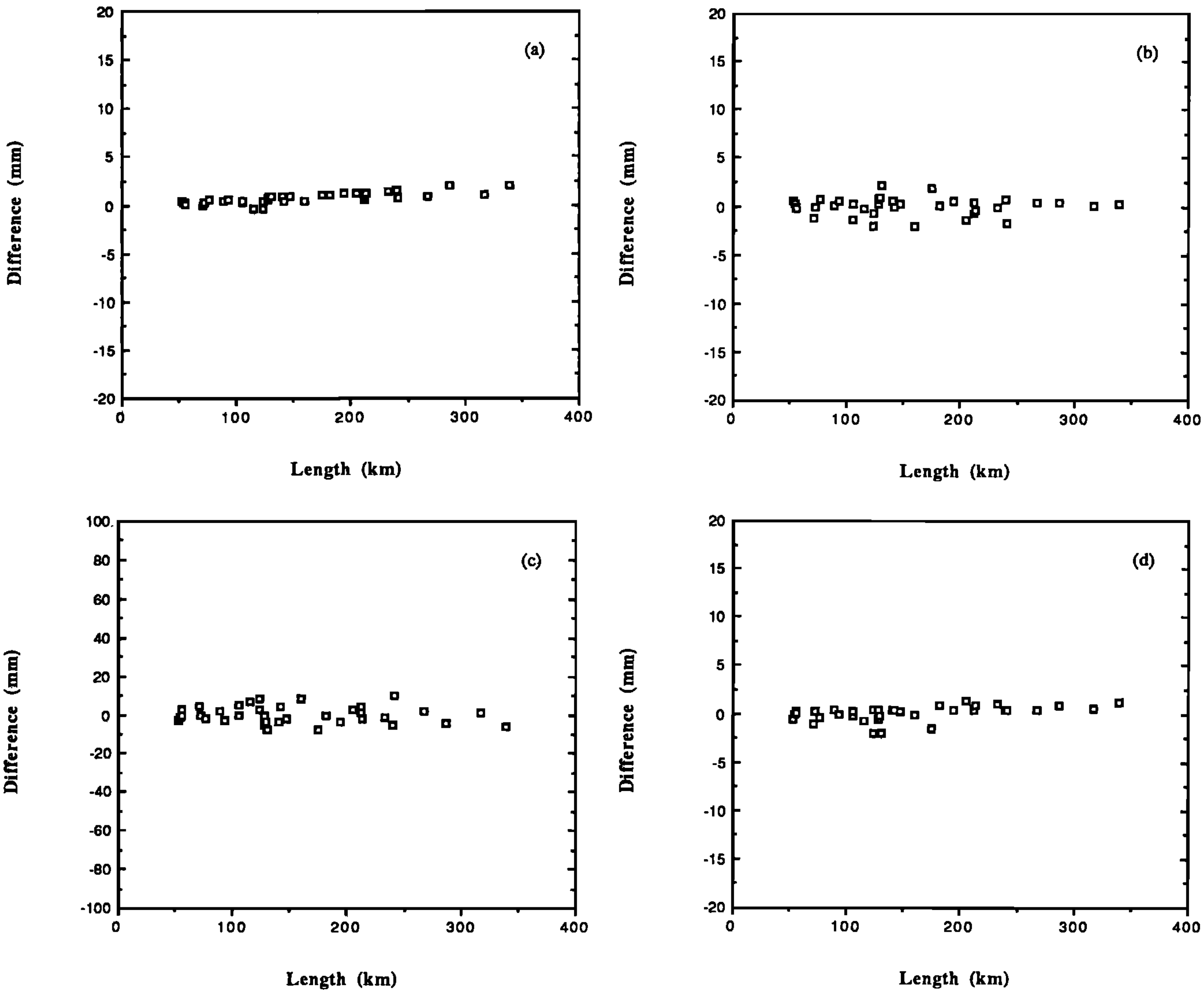

Fig. 2. Difference between interstation vectors in central California using two different fiducial networks: Westford-Richmond-OVRO and Algonquin-Richmond-Fort Ord. Zero is defined by the Westford- Richmond-OVRO mean solution, and the Algonquin-Richmond-Fort Ord is plotted relative to that. (*a*) north component. (*b*) east component. (*c*) vertical component. (*d*) length. Agreement between horizontal components is better than 2 mm. Vertical scatter is 110 mm. Dilation, as shown in Figure 2*d*, is small.

cial network for their analysis of data collected in southern and central California during January 1987: OVRO, Algonquin, and Platteville. They achieved very high precision. By fitting a line to their short-term precision estimate for interstation vectors, they report precision of 6 mm + 0.5 parts in 10^8 and 2.5 mm + 0.9 parts in 10^8 for the east-west and north-south components, respectively.

The short-term precision for this fiducial network is listed in Table 3 and shown in Figure 3, with the analogous precision for a geometrically strong network (Westford-Richmond-Fort Ord). By also showing Westford-Richmond-Fort Ord we see that the north-south precision for the Westford-Platteville-Fort Ord network is now length dependent, which contradicts the results for the "geometrically strong" fiducial networks. It is well known that block I GPS satellite tracks were preferentially aligned north-south over California (see Figure 2 in paper 1). For short-term precision, this resulted in better precision in the north-south component than the east-west component [*Blewitt*, 1989; paper 1]. Since the only factor we have changed for these data is the fiducial network we used (we cannot change the satellite geometry), it would appear that the geometry of the Westford-Platteville-Fort Ord fiducial network has influenced precision. Another indication that this is a suboptimal network is the vertical precision, which is substantially worse than that for Westford-Richmond-Fort Ord, and baseline dependent, as shown in Figure 3*c*. The vertical scatter reproduces another result of Dong and Bock: that vertical precision is dependent on baseline length using a fiducial network with poor geometry.

Although precision seems to be impacted by the geometry of the fiducial stations, the precision is still subcentimeter in the horizontal components of baselines less than 350 km in length. The more important question is whether the crustal deformation network has been rotated, sheared, dilated, or tilted relative to the solutions for a geometrically robust network. Figure 4 shows the Westford-Platteville-Fort Ord

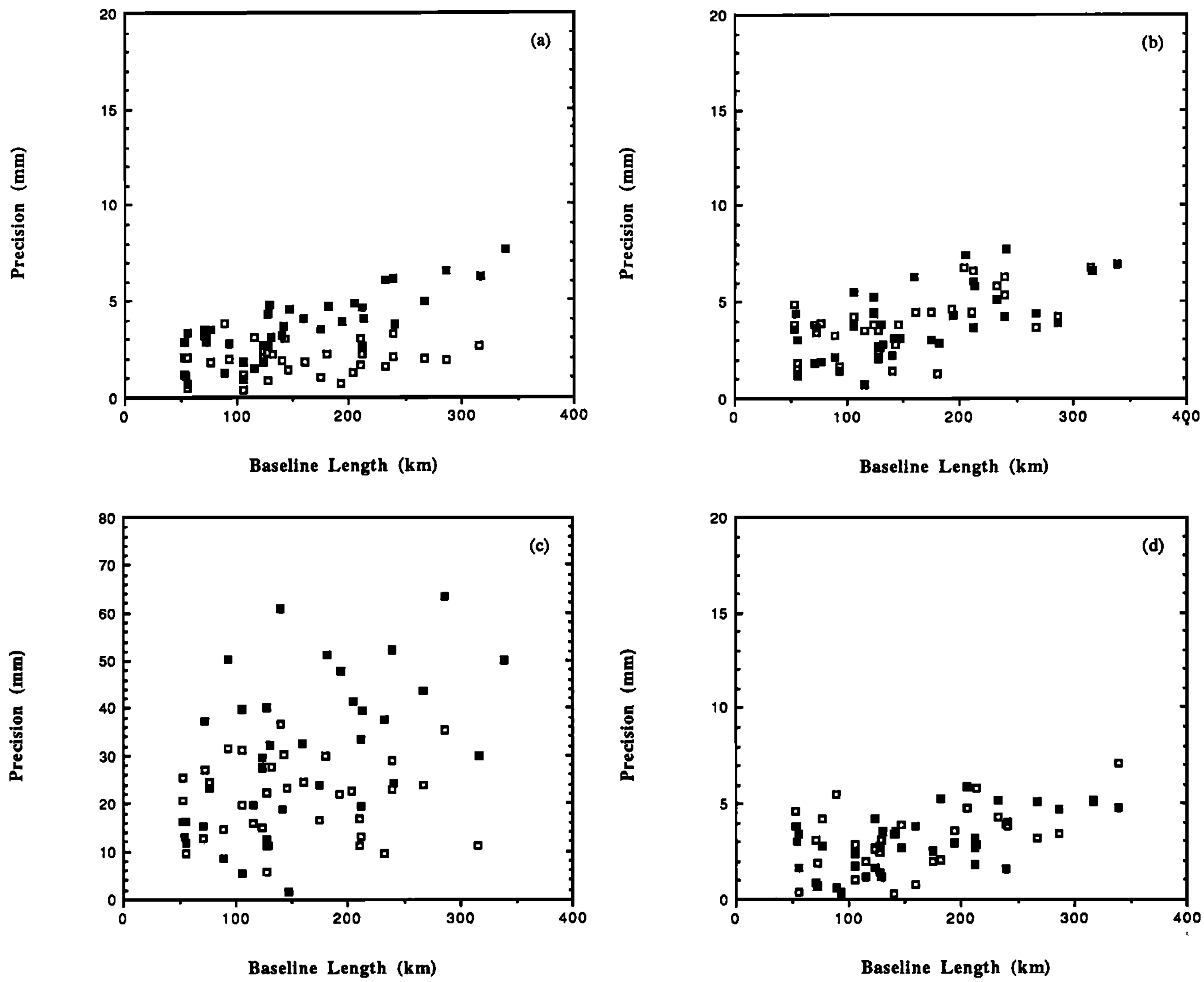

Fig. 3. Short-term precision, determined by the weighted RMS about the mean, for interstation vectors in the central California network for two fiducial networks: Westford-Richmond-Fort Ord, indicated by open squares and Westford-Platteville-Fort Ord, indicated by solid squares. The components are defined as in Figure 2. Note degraded precision of vertical for Westford-Platteville-Fort Ord fiducial network.

mean solution plotted relative to the Westford-Richmond-Fort Ord mean solution. The differences are an order of magnitude more than they were for the fiducial networks with "good" geometry. The pronounced difference in the length component is indicative of dilatation, which is also shown in the strain calculation. The dilatation for Westford-Platteville-Fort Ord network relative to the Westford-Richmond-Fort Ord network is 7.9×10^{-8}. The rotation is -1.6×10^{-8} rad. This rotation would produce an error of only 1.6 mm on a 100-km baseline but combined with the dilatation result, yields a 9.5-mm error on the same baseline vector. The maximum tilt is 2.1×10^{-7} rad, which is also an order of magnitude larger than maximum tilts for fiducial networks with good geometry. *Lichten et al.* [1989] found similar deterioration of accuracy for a baseline estimated with a geometrically deficient fiducial network of Hatcreek, Fort Davis, and Richmond. It might be argued that the data collected at Platteville were of poor quality. The post-fit residual RMS values for Platteville are equal to or better than several other fiducial sites. Also, more data were collected at Platteville than any other fiducial site. It might also be argued that the VLBI coordinates of Platteville are poorly known. This is not indicated by the formal errors in Table 1. These statistics indicate that geometry is the controlling factor, not quality or quantity of data collected at Platteville.

A large subset of the GPS data collected in southern and central California in paper 1 suffered from a more severe problem than poor fiducial geometry: simultaneous failure of a large percentage of tracking sites in North America. These data were analyzed with a fiducial network defined by VLBI sites in California. Therefore it is necessary to address the validity of fiducial networks which are defined on the same spatial scale as the crustal deformation network. Although one generally thinks of fiducial networks as merely a means to achieve some measure of orbit improvement, it is important to recall that these fiducial networks are also necessary for comparing repeated relative measure-

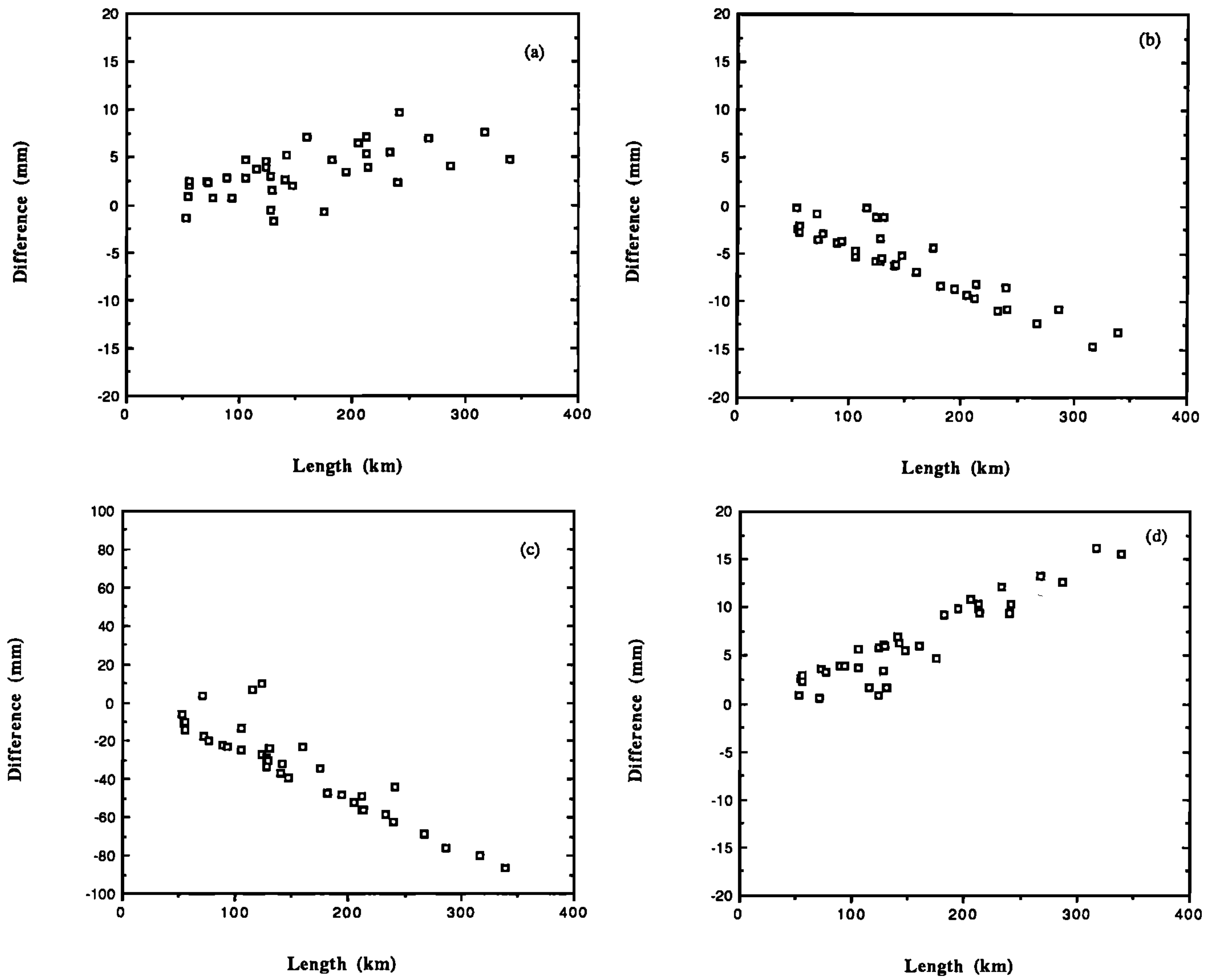

Fig. 4. Difference between interstation vectors in central California using two different fiducial networks: Westford-Platteville-Fort Ord and Westford-Richmond-Fort Ord. Zero is defined as the Westford- Richmond-Fort Ord mean solution, and the Westford-Platteville-Fort Ord mean solution is plotted relative to that. The components are defined as in Figure 2. Compared to Figure 2, where geometrically robust networks were compared, the horizontal component differences are nearly an order of magnitude greater. The vertical component differences are nearly 100 mm at 350 km. Note also the clear scale change for length (Figure 4*d*), which is indicative of areal dilatation.

ments. At best, the maximum length of fiducial baselines between California VLBI sites is 500 km. In addition to the rotation which might be caused by fixing sites on these spatial scales, inaccurate orbit solutions can adversely influence one's ability to resolve ambiguities (M. Murray, personal communication, 1989). So-called "local fiducial networks" are implicitly contained in the VLBI reference frame, as the self- consistent VLBI solution ties these sites in California to the continental solutions for Westford and Richmond. Yet, recalling the short discussion on the introduction of error or uncertainty in the position of a fiducial site, a several centimeter fiducial error on 500-km length scales would have an order of magnitude larger effect on interstation vector estimates than would a comparable error on a fiducial network defined over continental scales. If the locations of "local" fiducial sites should not be fixed, how tightly should local networks be constrained to their a priori locations? Do local networks achieve the same mm level precision and stability as Algonquin-Richmond-Fort Ord and Westford-Richmond-OVRO?

There were five VLBI sites occupied in California during the March 1988 experiment: Mojave, Fort Ord, OVRO, Vandenberg, and Palos Verdes. We tried three kinds of estimation strategies for this network. In each we fixed the position of Mojave. Then, we (1) estimated the positions of the other four sites with a 40-mm standard deviation in each coordinate, (2) estimated the positions of the other four sites with a 20-mm standard deviation in each coordinate, and (3) fixed all five fiducial sites. When the fiducial sites were estimated with a standard deviation of 40 mm, strategy 1, the baseline dependence of the scatter for the east-west component was 1.6 mm + 5.9 parts in 10^8, and 3.8 mm + 1.4 parts in 10^8 for a 20 mm standard deviation. Plots of precision for strategies 1 and 2 are shown in Figure 5. In the north-south component, 20 and 40 mm yield similar results, with a baseline dependence of less than 1 part in 10^8.

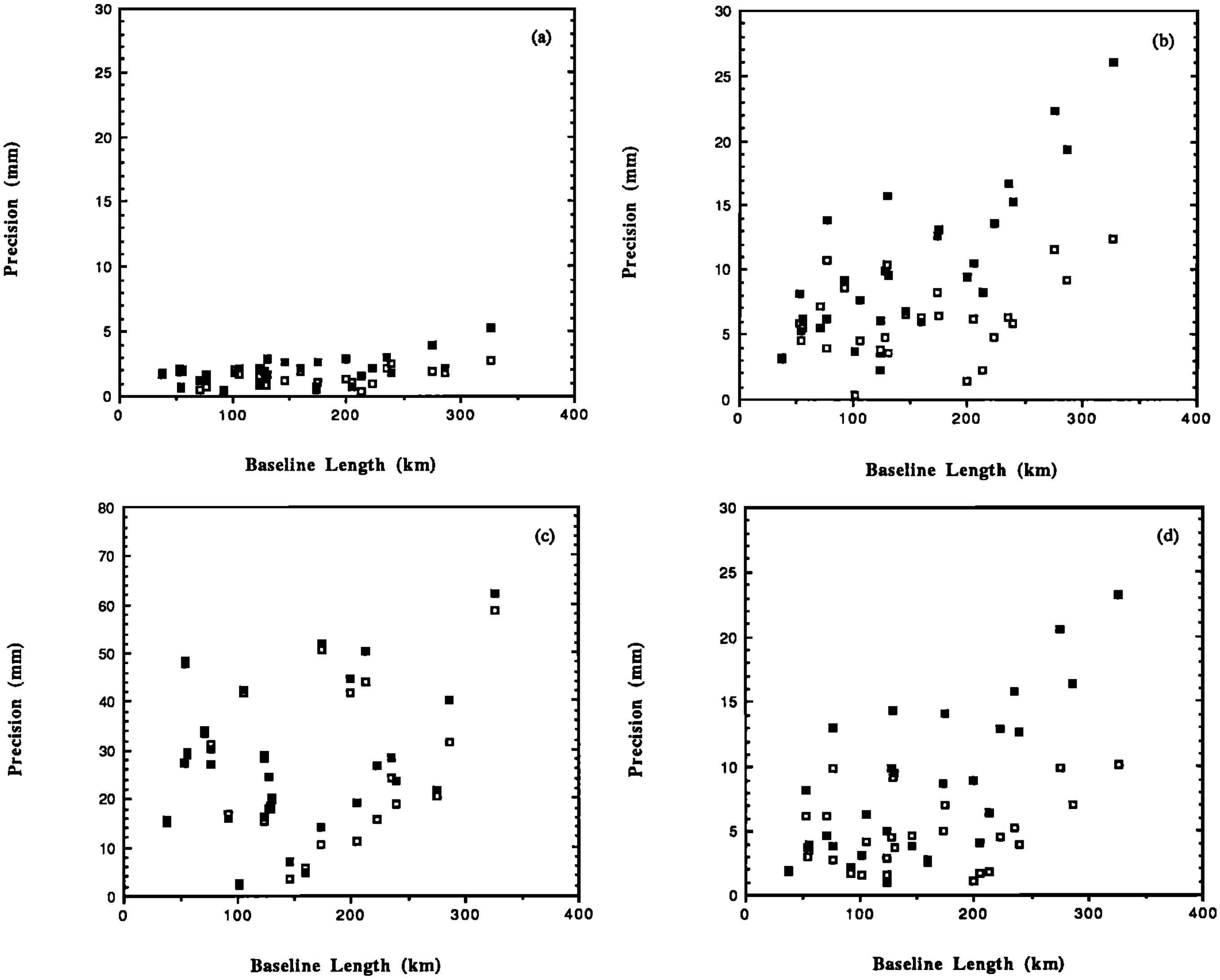

Fig. 5. Short-term precision, determined by the weighted RMS about the mean, for interstation vectors in the central California network for a fiducial network comprised of Mojave-OVRO-Palos Verdes- Vandenberg-Fort Ord. The open squares represent the case where fiducial sites were estimated with a standard deviation of 20 mm; the solid squares were estimated with a standard deviation of 40 mm. The components are defined as in Figure 2.

Given the satellite geometry over California, this result is not surprising. Vertical precision is markedly improved with a 20 mm standard deviation, as shown in Table 3. When we fixed all California VLBI sites, we found that the solution for the crustal deformation network differed only slightly from that computed with fiducial stations constrained to 20-mm, although the "all-fixed" solution was slightly less precise. Since a 20 mm standard deviation allowed us to compute meaningful formal errors for the interstation vectors in the crustal deformation network, we preferred this solution to the "all-fixed" solution, which we discuss later in this paper. As shown in Table 4, the strain analysis indicates that the Mojave-OVRO-Palos Verdes-Vandenberg-Fort Ord fiducial network dilates, shears, and rotates the crustal deformation network. Figure 6 shows the Mojave-OVRO-Palos Verdes-Vandenberg-Fort Ord mean solution plotted relative to the Westford-Richmond-Fort Ord mean solution. The dilatation for this network is -8.3×10^{-8}, the largest of all fiducial networks we studied. The rotation is -3.2×10^{-8} rad. Although Table 3 indicates that the north-south component is precise, Figure 6 indicates that the component has contracted, relative to the VLBI reference frame. The maximum tilt is significantly smaller than when we used the Westford-Platteville-Fort Ord fiducial network, only 5.3×10^{-8} rad. Presumably, constraining so many sites in California provides sufficient geometric strength to determine the vertical component.

We have also tested the effect of constraining only three VLBI sites in California: Mojave, Vandenberg, and Palos Verdes (this type of fiducial network was used in experiments J87 and M89a in paper 1). This network does not behave significantly differently than the five-station California fiducial network, although the rotation is slightly larger, which is consistent with a having fewer fiducial stations for geometric constraint, and a larger apparent shear. The dilatation of Mojave-Palos Verdes-Vandenberg is smaller than Mojave-OVRO-Palos Verdes-Vandenberg-Fort Ord, -3.5×10^{-8}, and the rotation is -5.3×10^{-8} rad. The maximum

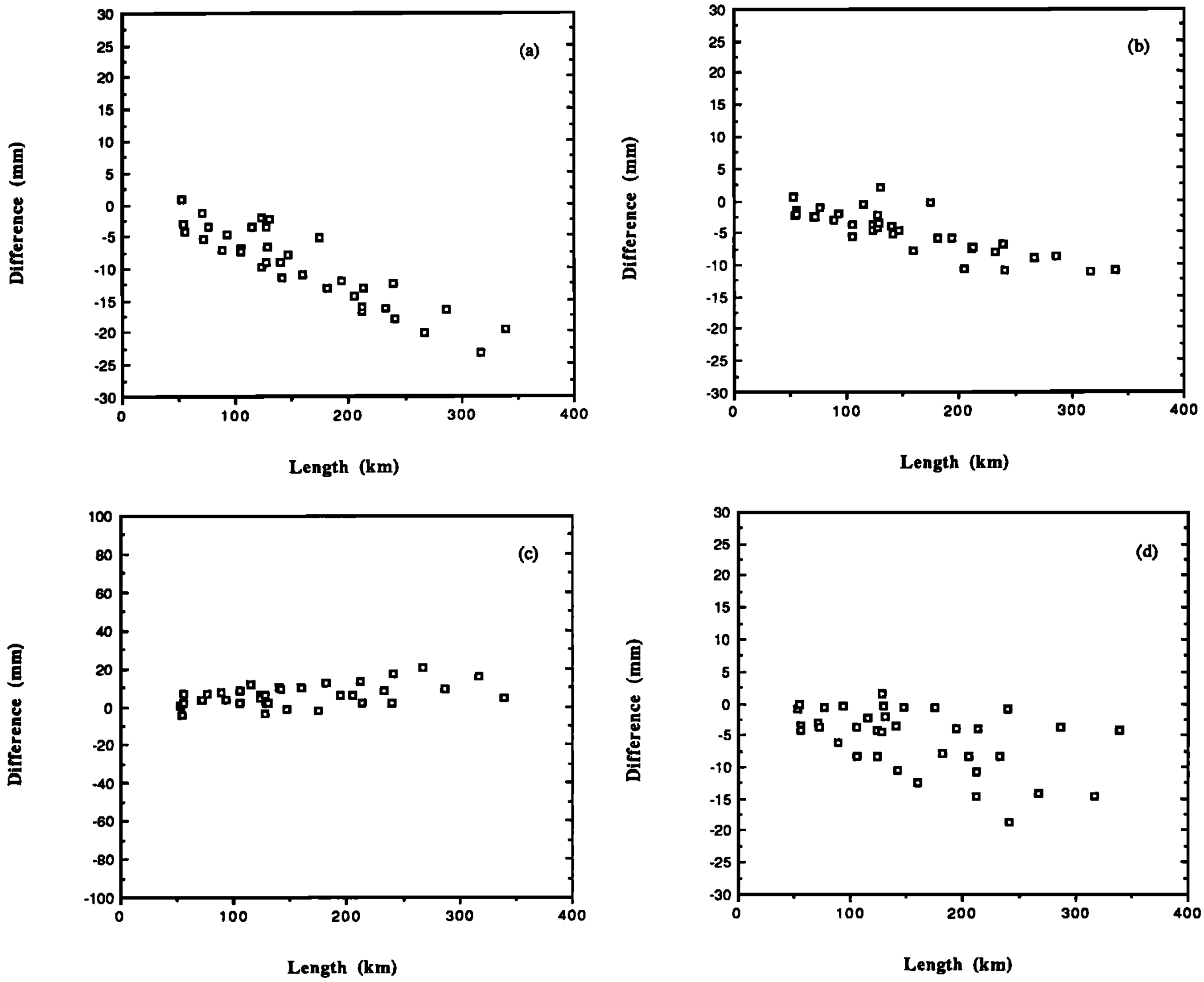

Fig. 6. Difference between interstation vectors in central California using two different fiducial networks: Mojave-OVRO-Palos Verdes- Vandenberg-Fort Ord and Westford-Richmond-Fort Ord. Zero is defined as the Westford-Richmond-Fort Ord mean solution, and the Mojave-OVRO-Palos Verdes-Vandenberg-Fort Ord mean solution is plotted relative to that. The components are defined as in Figure 2.

tilt, 1.8×10^{-7} rad, is twice as large as Mojave-OVRO-Palos Verdes-Vandenberg-Fort Ord, as might be expected given that there would be less of a vertical constraint from three fiducial sites than five.

Discussion

We have shown that interstation vector estimates change considerably, depending on the fiducial network one uses. We think that these changes are consistent with the geometry and spatial scale of the fiducial stations. If "deficient" fiducial networks must be used, we need some way to represent this systematic error in our formal errors. One way to compensate for changes in fiducial network is by determining the sensitivity of interstation vectors in the crustal deformation network to uncertainties in fiducial coordinates. Such a sensitivity analysis, sometimes called a "consider" analysis, is dependent on the assumed uncertainties for the fiducial coordinates. Alternatively, one can estimate fiducial positions, with the appropriate weighted standard deviation. We briefly discuss these options, focusing on the results for the Westford-Platteville-Fort Ord and Mojave-OVRO-Palos Verdes-Vandenberg-Fort Ord fiducial networks, discuss concerns for the choice of fiducial sites, and present an example of the effect of fiducial bias on the determination of crustal deformation rates.

Sensitivity Analysis

Could we have predicted the effect that changing fiducial networks had on our crustal deformation network? The magnitude of any systematic error can be estimated by performing sensitivity analyses. By assigning plausible uncertainties to unadjusted parameters, one determines the sensitivity of estimated parameters, explicitly accounting for correlations through a full covariance analysis. Thus sensitivities are calculated from the measurement partials and geometry. This technique is used and described by *Lichten and Border* [1987]. We have computed the consider uncertainty for two fiducial networks, Westford-Richmond-Fort

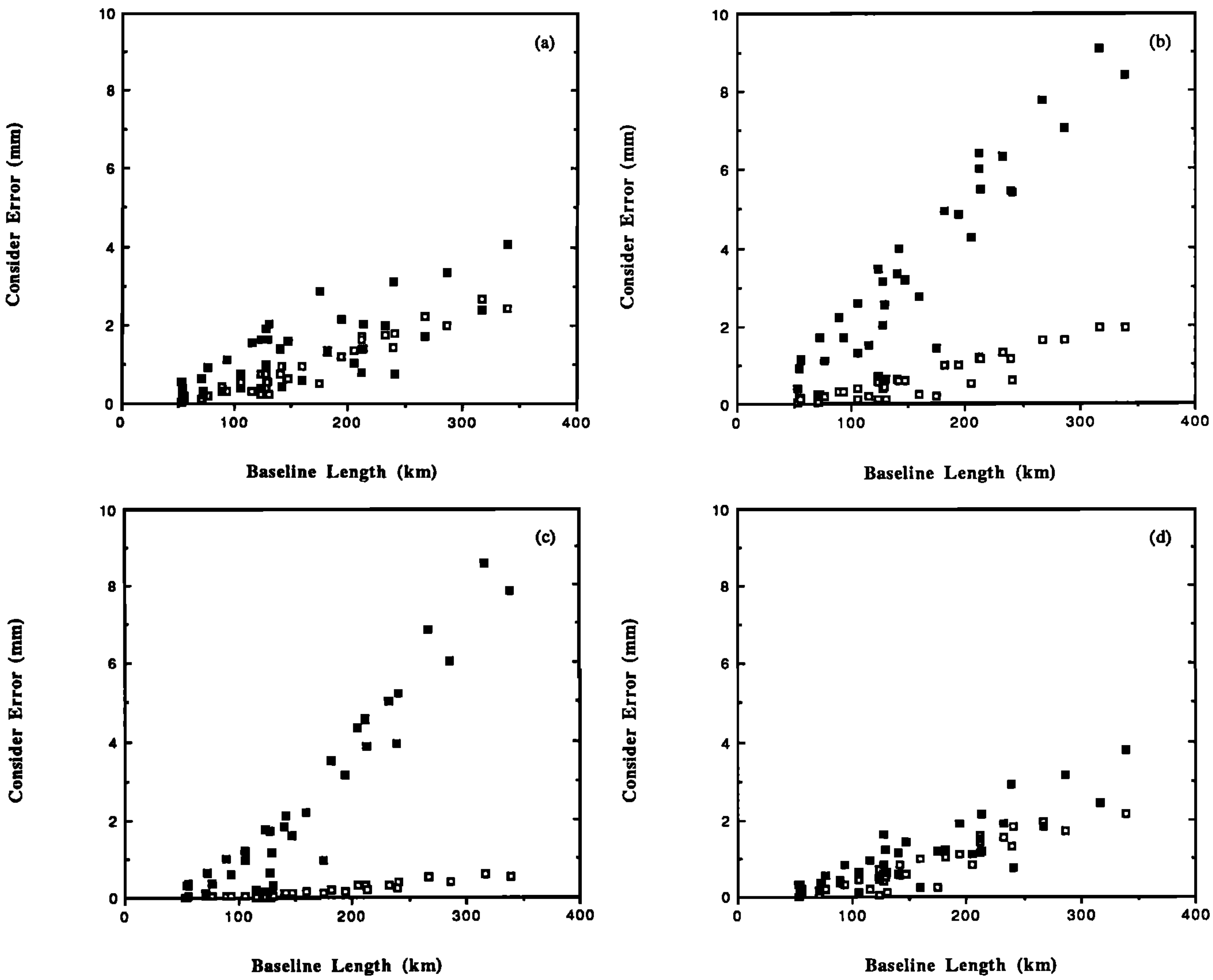

Fig. 7. Computed consider errors for the crustal deformation network, using fiducial networks: Westford-Platteville-Fort Ord (solid squares) and Westford-Richmond-Fort Ord (open squares). The components are defined as in Figure 2. Computation of consider errors described more fully in the text.

Ord and Westford-Platteville-Fort Ord. Each fiducial site was assigned an uncertainty of 20 mm in all three components. Although Lichten and Border [1987] have suggested more conservative errors for fiducial sites (up to 40 mm), we feel that the results of Lichten and Border and other accuracy analyses [*Blewitt*, 1989; paper 1], are more consistent with 20-mm fiducial uncertainty. There are certainly other systematic errors which have not been accounted for in the GPS error budget, but since we are only trying to determine the error contribution from fiducial coordinates, we prefer the smaller value. No correlations between the fiducial sites were used. The total error σ_{total} is then

$$\sigma^2_{total} = \sigma^2_{consider} + \sigma^2_{formal} \tag{18}$$

Our formal errors are described in paper 1. We first examine the fiducial (consider) contribution to the interstation vector estimate error, shown in Figure 7. We have plotted consider errors for both Westford-Richmond-Fort Ord and Westford-Platteville-Fort Ord, so that we can compare and contrast their effect on the crustal deformation network. For a geometrically "strong" network such as Westford-Richmond-Fort Ord, the horizontal consider error is small, less than 2 mm at 350 km, with little distinction between north- south and east-west components. The maximum vertical consider error using Westford-Richmond-Fort Ord is 1 mm. In contrast, the consider error for Westford-Platteville-Fort Ord is highly sensitive in the east-west and vertical components, with a baseline dependence of 3 parts in 10^8. The consider length error is not as pronounced as we saw with the actual data. The consider covariance analysis is consistent with a rotation between the east-west and vertical components but does not predict dilatation.

Are the consider errors sufficient to explain the differences we observed between Westford-Platteville-Fort Ord and strong geometry networks? In Figure 8, we plot the total error, as defined in equation (18), for the Westford-Platteville-Fort Ord solution, along with the offset of these estimates from the solutions computed with Westford-Richmond-Fort Ord (these offsets are also shown in Figure 4). In both north-south and east-west components, the modi-

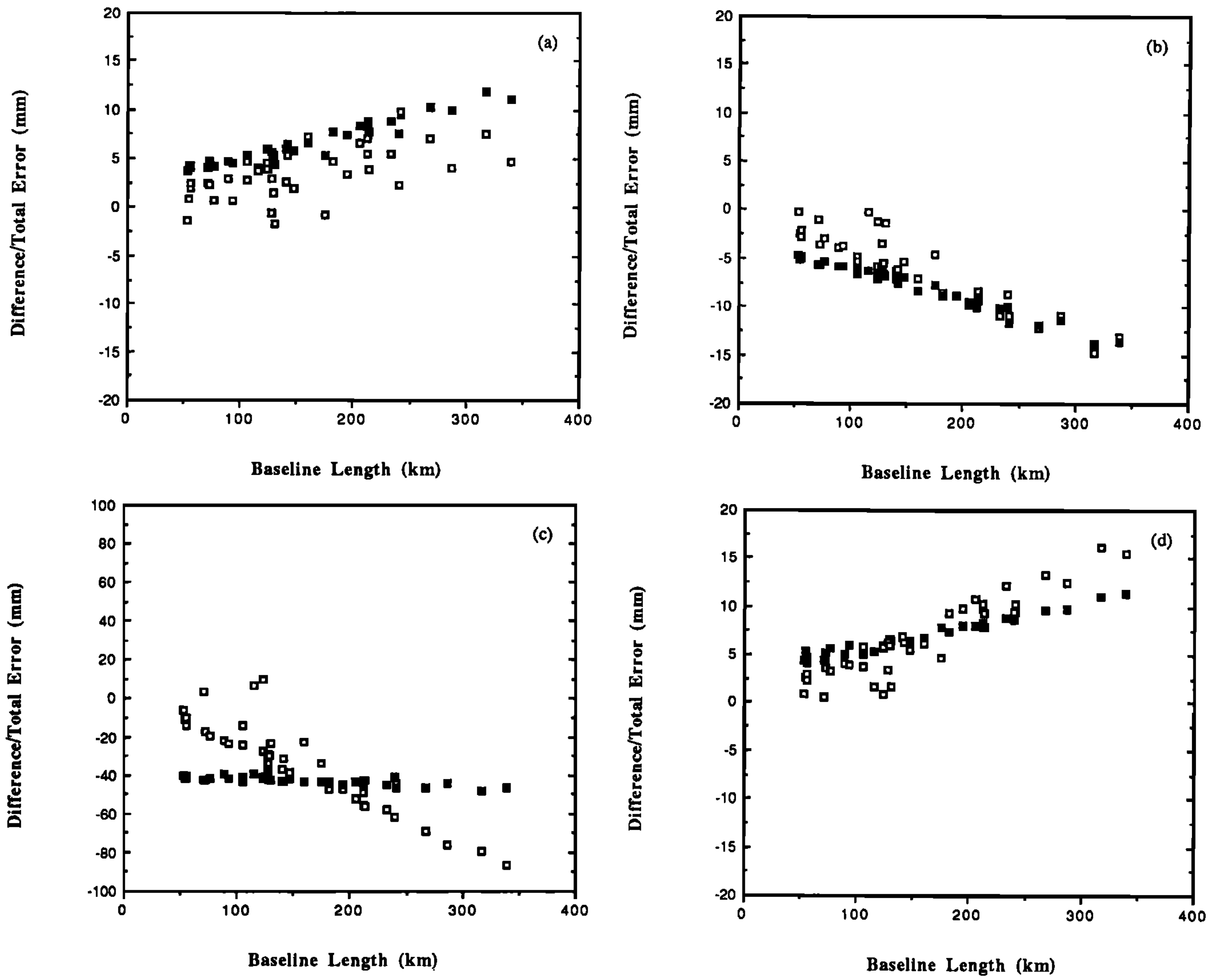

Fig. 8. Computed total errors for the crustal deformation network (solid squares) using fiducial network Westford-Platteville-Fort Ord. Also shown, offset of crustal deformation network interstation vectors from those computed using fiducial network Westford-Richmond-Fort Ord (open squares). These offsets were also shown in Figure 4. Whereas the offsets in the north-south and east-west components can be described by the formal errors, modified by fiducial consider errors, the vertical component and length are more highly degraded than expected. The components are defined as in Figure 2.

fied total errors are just large enough to describe the offset. In the vertical component, the total error does not have the pronounced baseline dependence we saw in the actual data. The length plot follows directly from the component results.

Estimation of Fiducial Coordinates

It is possible to estimate the positions of fiducial sites from GPS data. The standard deviations for the position uncertainties will then be reflected in the formal uncertainties of the interstation vectors. Unfortunately, by estimating fiducial locations, the reference frame will change, although only slightly, depending on the accuracy of the fiducial coordinates. Since we need to maintain a stable reference frame for comparing repeated measurements, we discourage estimating fiducial coordinates. In the one case we have estimated fiducial coordinates, the fiducial network Mojave-OVRO-Palos Verdes-Vandenberg-Fort Ord, the formal errors for the crustal deformation network interstation vectors were larger than for those estimated with Westford-Richmond-Fort Ord. Figure 9 displays the formal errors we have calculated for the interstation vectors in our crustal deformation network, using the fiducial networks Mojave- OVRO-Palos Verdes-Vandenberg-Fort Ord and Westford-Richmond- Fort Ord. While the vertical component formal error (Figure 9*c*) shows little dependence on which fiducial network we used, both horizontal components show a strong deterioration with baseline length when a "local" fiducial network was used, described by several mm + 4 parts in 10^8. The Westford-Richmond-Fort Ord network has formal errors in keeping with the short-term precision we calculated in paper 1, several millimeters + 0.5-1 parts in 10^8. The differences between interstation vectors computed from these two fiducial networks were shown in Figure 6. When we estimate the position of fiducial stations in a local network, the formal errors are large enough to explain the observed offsets from a stable fiducial network, such as Westford- Richmond-Fort Ord.

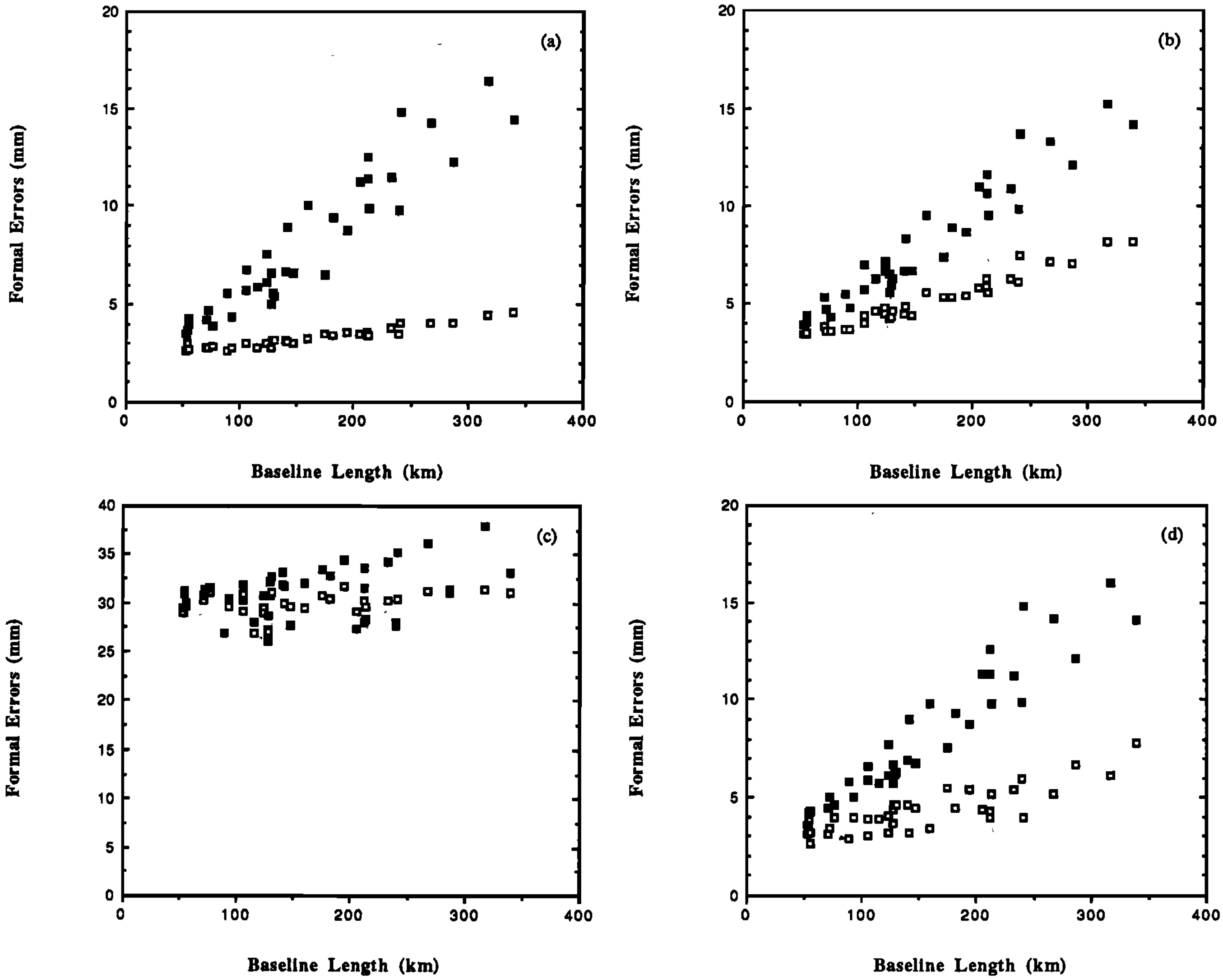

Fig. 9. Formal errors for interstation vectors in the crustal deformation network, using fiducial networks: Mojave-OVRO-Palos Verdes-Vandenberg-Fort Ord (solid squares) and Westford-Richmond-Fort Ord (open squares). Note the deterioration with baseline length for the north-south and east-west components using the Mojave-OVRO-Palos Verdes-Vandenberg-Fort Ord fiducial network. Vertical components (Figure 9*c*) are only slightly affected by these fiducial networks. The components are defined as in Figure 2.

Effect of Fiducial Biases on Crustal Deformation Rates

We are interested in applying GPS to crustal deformation measurements, and therefore, our underlying concern is that fiducial biases, such as we have described, will contaminate our determination of vector rates. The safest way to avoid bias of rate determinations is to use the same kind of fiducial network at each epoch of the measurement history, where the geometry of fiducial stations has been taken into account. If that kind of fiducial network is unavailable, it is possible that a fiducial bias, if common to all epochs, will not effect rates. An error in the geocenter correction is an example of such a fiducial bias, affecting interstation vectors equally at each epoch, and thus would be unimportant for rate determinations.

The effect of fiducial biases can be seen in some of the interstation vectors we estimated in paper 1. Because we have data spanning several years, we can also calculate the effect of a suspected fiducial bias on our crustal deformation rates. Figure 10 displays the north-south component of the interstation vector between OVRO and Blackhill, a baseline 308 km long. The first 13 measurements were made over four epochs, spaced approximately 5 months apart. The final three measurements were made 1 year later. The first four epochs were analyzed with continental-scale fiducial networks, with fairly good geometry (see Table 2 of paper 1 for details of these experiments). The final experiment (M89b) used a fiducial network of five VLBI sites in California. Recall that in our analysis of this type of fiducial network, there was an apparent strain and rotation of nearly 1 part in 10^7. The north-south component was particularly affected. The error bars for the M89b estimates are from 2 to 4 times larger than those computed with a continental scale fiducial network, because we have estimated the fiducial sites with a standard deviation of 20 mm (as described in the previous section). The three lines shown in Figure 10 are where we used data from all 5 epochs, weighted by the formal errors shown (solid line), only used the data from the first four epochs, weighted by the formal errors shown (dotted line), and used data from all five epochs, but the

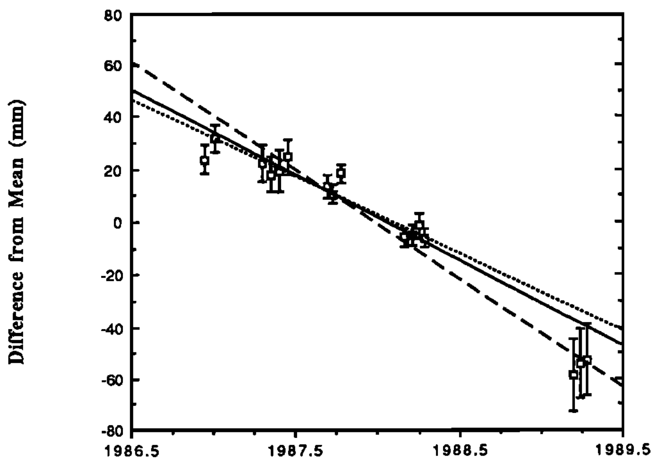

Fig. 10. Change of relative coordinates for the north-south component of the Blackhill to OVRO vector (308 km), with zero being the mean value. Also shown are three linear fits to the data. The solid line uses all data, weighted by the formal errors. The dotted line fit excludes the data from the last experiment (March 1989), and the dashed line uses all data, but the formal errors in the March 1989 estimates were not modified to take fiducial uncertainty into account. The estimates of slope vary from -33±3, -30±3, and -42±2 mm/yr using these three sets of data. More discussion in the text.

formal errors for the fifth experiment were not adjusted to take into account the poor geometry of the California-only fiducial network. Therefore the errors bars are based on data noise only; that is, the fiducial coordinates were held fixed in the estimation of interstation vectors, and no consider analysis was performed (dashed line). Linear fits to these three data sets result in slopes of -33±3, -30±3, and -42±2 mm/yr, with χ^2 of 1.1, 0.9, and 2.3, respectively (χ^2 defined in paper 1). Although the slopes for the first two data sets are slightly different, the agreement is far better than when one makes no consideration of fiducial bias, as in the last case. The difference between the linear fit excluding epoch 5 (dotted line) and the data from epoch 5 is 25 mm, which agrees with the expected difference for a 300-km baseline shown in Figure 6*a*. Thus fiducial biases can significantly affect GPS rate determinations.

How to Choose a Fiducial Network

Most GPS experiments are driven by cost considerations. Therefore it is most cost effective to use existing permanent, continuous networks, such as CIGNET. Although we have found that three fiducial sites are adequate for precise measurements, our work has been focused on California, where the block I GPS constellation was favorably oriented during our experiments. Others have suggested that four fiducial sites may be appropriate in North America [*King and Blewitt*, 1990]. For experiments which span continental distances, a tracking network which spans 10,000 km may be appropriate. Large tracking networks may also be necessary when the GPS constellation geometry is poor. During the CASA UNO experiment in South America, the block I constellation was visible for many hours, but the satellites were at low elevation angles, thus degrading accuracy and precision [*Freymueller and Golombek*, 1986]. When *Kornreich-Wolf et al.* [1990] examined data from that experiment, they found that extending the tracking network to Europe and Australasia improved orbit estimation and thus baseline estimates in South America. Therefore the number and location of fiducial sites depend on the geometry of the crustal deformation experiment and the GPS constellation. Although the profusion of autonomous receivers makes it likely that global fiducial networks will be available to all GPS users, it is important to note that mixing of different types of GPS receivers has yet to be shown to be valid for centimeter level accuracies required for crustal deformation experiments.

We have used VLBI derived fiducial positions because they are accurate, easily available (i.e. CIGNET stations are located near VLBI sites) and self-consistent [*Clark et al.*, 1987]. This does not mean that VLBI (or SLR) measurements are the only source of fiducial stations. Accuracies of several centimeters have been reported from GPS over long continental-scale baselines [*Lichten and Border*, 1987], and since these GPS coordinates were derived in a VLBI reference frame, it should be possible to use GPS derived "fiducial" coordinates.

One indication we have that GPS can be used for fiducial coordinates comes from the experiment (M88b) used in this paper. Basically, if the estimated GPS coordinates agree with the a priori VLBI coordinates, within some acceptable limit, then we have sufficient accuracy that we can use the coordinates of the other GPS stations, which were not collocated with VLBI sites, as fiducial stations. Table 5 lists the offsets of three GPS stations, OVRO, Algonquin, and Mojave, from their expected VLBI coordinates. We used a fiducial network of Westford-Richmond-Fort Ord to compute these estimates and to report the weighted mean solution for 3 days of data. The uncertainties are one standard deviation. We did not compute consider uncertainties for these experiments. The GPS-VLBI offsets for Algonquin, Mojave, and OVRO range from 6 to 40 mm in magnitude in the three Cartesian components. Let us assume that the VLBI coordinates are perfect and that these offsets represent GPS error. Our consider covariance analysis can then be used to determine the effect of these GPS "errors" on the interstation vectors in the crustal deformation networks. We recomputed solutions for the crustal deformation network, using three separate fiducial networks: Algonquin-Richmond-Fort Ord, Westford-Richmond-Mojave, and Westford-OVRO-Richmond. The fiducial coordinates for all sites were derived using the VLBI model as before, but the consider uncertainties for Algonquin, Mojave, and OVRO were adopted from the uncertainties from Table 5. For consistency, we used a 20-mm consider error for the other two fiducial sites in each fiducial network. Table 6 lists the consider uncertainties we used, and the computed consider uncertainty in the crustal

TABLE 5. Offset of GPS Estimates From VLBI Coordinates

	X, mm	*Y*, mm	*Z*, mm
Algonquin	32±10	-6±22	-7±18
Mojave	-14±12	-17±21	-10±16
OVRO	36±20	40±23	-30±18

Solution computed with Fort Ord, Westford, and Richmond fixed. Uncertainties are unscaled standard deviations.

TABLE 6. Consider Analysis

Fiducial Network	Assumed Fiducial Uncertainty, mm			Computed Consider Uncertainty, 10^{-8}			
	X	Y	Z	East	North	Vertical	Length
Algonquin	32	6	7	0.7	0.9	0.8	0.8
Richmond	20	20	20				
Fort Ord	20	20	20				
Mojave	14	17	10	0.6	1.2	1.2	1.0
Westford	20	20	20				
Richmond	20	20	20				
OVRO	36	40	30	0.7	1.2	1.6	1.0
Westford	20	20	20				
Richmond	20	20	20				

Fiducial uncertainty of Algonquin, Mojave, and OVRO taken from Table 5. All other fiducial uncertainties set to 20 mm in each coordinate.

deformation network, as a function of baseline length. Even for OVRO, where offsets from VLBI are as large as 40 mm, the effect on the crustal deformation network is small, 1.2 parts in 10^8 in the north-south component, slightly larger in the vertical component, 1.6 parts in 10^8. These fiducial consider uncertainties are the same magnitude as we estimated from our strain calculations for different fiducial networks. Thus GPS-derived fiducial coordinates provide a comparable level of stability as VLBI. Single epoch estimates of GPS fiducial coordinates will not be as accurate as VLBI estimates which have been measured over many years, but they are an acceptable alternative.

We have also tested the use of GPS-determined fiducial coordinates. We estimated the fiducial coordinates of Churchill, Manitoba, using data from M88b. We cannot use our standard strain analysis technique to check a fiducial network consisting of, for example, Westford-Churchill-Fort Ord, since the coordinates of Churchill are consistent (in a least squares sense) with the fiducial coordinates and the data which were used to compute it. In order to validate the use of GPS-derived Churchill coordinates, we need to use the Churchill fiducial coordinates in an independent GPS survey. Paper 1 and *Larson* [1990*b*] used Churchill as a fiducial site during experiments D86, S87, M88a, adopting the coordinates determined from M88b. We observed no difference, larger than a part in 10^8, between those estimates which used Churchill as a fiducial, and those that did not. This is indirect evidence that GPS can be used to supplement the number of available fiducial stations. As an example of the agreement we are taking about, the first and third epochs shown in Figure 10 were determined using Churchill as a fiducial site. Note the agreement of the straight-line fit with the data from the second and fourth experiments, where Churchill was not used.

CONCLUSIONS

By comparing the interstation vector estimates for a crustal deformation network in central California, spanning approximately 350 km, where we have varied the fiducial networks, we have shown that three receivers collocated with VLBI sites provide a stable reference system at the several millimeter level in the horizontal and at 10 mm in the vertical components. This result implies that local survey errors between VLBI and GPS monuments at fiducial sites do not severely affect the estimation of interstation vectors. The geometry of the three fiducial sites must be taken into account if a stable reference frame is to be obtained. When the three fiducial stations all lie close to a great circle, strains and rotations as great as a part in 10^7 are produced. If there are not a sufficient number of VLBI sites in one's experiment, GPS-derived fiducial coordinates can be used. Of course, these coordinates should be determined in the same kind of reference frame, such as that defined by VLBI. Fiducial networks which span the same scale as the crustal deformation network dilate and rotate the crustal deformation network by a part in 10^7 and are not suitable for crustal deformation networks over 20 km in scale.

The bias that fiducial networks produce on crustal deformation networks can be mitigated somewhat by use of a consider covariance analysis. Although the adopted uncertainties for fiducial coordinates is somewhat arbitrary, this technique does allow the total error to reflect fiducial bias, in a way that reduces its affect on measurement of crustal deformation rates. Fiducial coordinates may be estimated, which would also increase the interstation vector standard deviations in the crustal deformation network, but this would also allow the reference frame to change, producing unwanted dilatation, rotation, and tilt.

The requirement for a fiducial network is most easily met if there is a preexisting, reliable, continuously operating network dedicated to precise orbit determination. If one's geodetic network includes baselines longer than 20 km, this strongly suggests that if such a tracking network does not exist or is not reliable, it is in one's best interest to create a fiducial network for the duration of the experiment. By sacrificing mobile sites in the crustal deformation network, those receivers and field operators can be sent to VLBI sites, thereby providing the data needed for precise orbit determination and a stable reference frame.

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