

An assessment of relativistic effects for low Earth orbiters: the GRACE satellites

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Abstract

The GRACE mission consists of two identical satellites orbiting the Earth at an altitude of ~ 500 km. Dual-frequency carrier-phase Global Positioning System (GPS) receivers are flying on both satellites. They are used for precise orbit determination and to time-tag the K-band ranging system used to measure changes in the distances between the two satellites. The satellites are also flying ultra-stable oscillators (USOs) to achieve the mission's need for short-term (< 1 s) oscillator stability. Because of the high quality of both the GPS receivers and the oscillators, relativistic effects in the GRACE GPS data can be examined. An expression is developed for relativistic effects that explicitly includes the effects of the Earth's oblateness (J_2). Use of this expression significantly reduces the twice per orbital period energy in the GRACE clock solutions, indicating that the effect of J_2 can be significant and should be modeled for satellite clocks in low Earth orbit. After relativistic effects have been removed, both GRACE USOs show large (2 ns to 3 ns) once per orbital period signatures that correlate with voltage variations on the spacecraft.

1. Introduction

While best known as a system that makes it possible to determine one's location on the Earth, the Global Positioning System (GPS) is also an important contributor to precise time- and frequency-transfer systems. Since signals from the GPS satellites themselves are linked to atomic frequency standards, a GPS user can determine the offset of his or her timepiece in real time with respect to GPS time (as defined by Space Operations Command). The accuracy of this timing estimate will be influenced both by the number of satellites being tracked by the GPS receiver and by their geometry in the sky. Timing accuracy is also limited by uncertainties in the GPS measurements, one's ability to model the location of the satellite transmitters and receiving antennae, and removal of delays due to the atmosphere (troposphere and ionosphere) and relativity. Previous studies of GPS time-transfer techniques have been limited to clocks on the Earth, e.g. Schildknecht

et al (1990), Baeriswyl *et al* (1994) and Larson and Levine (1999). A more challenging GPS time-transfer environment exists for space-borne receivers such as those flown on the twin GRACE satellites. In addition to the more complicated modeling needed to properly use the measurements on an orbiting spacecraft, the relativistic effects on an orbiting clock are also more complicated. In this study, data from the GRACE GPS receivers are examined to evaluate whether standard relativistic corrections as used in the GPS are of sufficient accuracy for space-borne GPS time-transfer. In the following section of this paper, standard corrections are examined and compared with new expressions derived for low Earth orbiters. Subsequently these new relativistic expressions are used to analyse the GRACE GPS data.

2. Relativistic clock effects

Clocks in orbit experience relativistic frequency shifts (see, for example, Ashby and Spilker (1996)). These effects must be

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removed if one is to make a meaningful comparison between the time kept by a clock on a satellite and that kept by one on the ground. These frequency shifts have both constant and time-varying components. The time-varying components are primarily caused by the eccentricity of the satellite's orbit and the quadrupole (and higher) terms of the Earth's gravity field. Thus, the time-varying frequency shifts are periodic in nature, with frequencies of once per revolution, twice per revolution, etc (hereafter referred to as once/rev, twice/rev, etc). Because the frequency shifts have a periodic component, so will the resulting time corrections which must be applied to the satellite's clock.

Let τ represent the relativistic effect on the time kept by the satellite clock. In order to account for such effects the correction $-\tau$ must be added to the time; for a clock in a Keplerian orbit (see ICD-GPS-200c (1993)) the effect is

$$\tau = -\frac{2}{c^2} \sqrt{aGM} \cdot e \cdot \sin E, \quad (1)$$

where a , e and E are the osculating semi-major axis, eccentricity and the eccentric anomaly of a satellite. M is the Earth's mass, G is Newton's gravitational constant and c is the vacuum speed of light. An alternative formulation often used in precise geodetic applications is

$$\tau = -2 \frac{\mathbf{r} \cdot \mathbf{v}}{c^2}, \quad (2)$$

where \mathbf{r} and \mathbf{v} are satellite position and velocity vectors, respectively. Equations (1) and (2) involve keeping only the leading monopole part of Earth's gravitational potential. Such approximations, while nearly negligible at the high altitude of a GPS orbit, are less accurate if applied to a low Earth satellite such as GRACE. Therefore, we now develop a more exact expression for the correction of periodic relativistic effects that can be applied to a satellite in an arbitrary Earth-centred inertial (ECI) orbit.

Two relativistic effects affect the frequency of a clock. The first effect is time dilation, or the so-called 'second-order Doppler effect'. Compared with a clock at rest in the ECI frame, the fractional frequency shift of a clock moving with velocity \mathbf{v} is

$$\frac{\Delta f}{f} = -\frac{1}{2} \frac{v^2}{c^2}, \quad (3)$$

where v is the magnitude of \mathbf{v} .

The second relativistic effect is the gravitational redshift which may be expressed in terms of the Newtonian gravitational potential $\Phi(\mathbf{r})$ at the position \mathbf{r} of the clock. The Newtonian gravitational potential carries a negative sign, for example $-GM/r$. If Φ_0 is the gravity potential of a reference clock at rest on Earth's geoid, including the centripetal potential due to Earth's rotation, then the fractional gravitational frequency shift is

$$\frac{\Delta f}{f} = \frac{\Phi(\mathbf{r}) - \Phi_0}{c^2}. \quad (4)$$

Φ_0/c^2 was defined by IAU to be $-6.969\,290\,134\,3 \times 10^{-10}$ (IAU 1991). More recently it has been defined by fixing the relation between geocentric coordinate time (TCG) and terrestrial time (TT) to have a rate of $-6.969\,290\,134 \times 10^{-10}$

(IAU 2000). Adding equations (3) and (4) yields the net fractional frequency shift:

$$\frac{\Delta f}{f} = \frac{\Phi(\mathbf{r}) - \Phi_0}{c^2} - \frac{1}{2} \frac{v^2}{c^2}. \quad (5)$$

Consider a clock in the orbiting satellite (defined as the s clock) and a reference clock on the rotating Earth. Let τ_s be the proper time elapsed on the orbiting clock. Then

$$\tau_s = \int dt \left[1 + \frac{\Phi(\mathbf{r}) - \Phi_0}{c^2} - \frac{1}{2} \frac{v^2}{c^2} \right], \quad (6)$$

where the integration is performed over coordinate time t . An equivalent expression for proper time for a reference clock at rest on the Earth can also be defined; however, the non-unity terms in the integrand of (6) will be very small and will be due only to the clock's elevation above the geoid. Thus, we can compute the relativistic correction that must be applied in order to compare the performance of the space oscillator to that on the ground using only the satellite component given in (6).

In computing the gravitational potential, we include the quadrupole portion of the Earth's gravity field (denoted by coefficient J_2). Thus,

$$\Phi(\mathbf{r}) = -\frac{GM}{r} \left[1 - J_2 \left(\frac{a_1}{r} \right)^2 \frac{(3z^2 - r^2)}{2r^2} \right], \quad (7)$$

where a_1 is the equatorial radius of the Earth and z is the third Cartesian component of the position vector \mathbf{r} in the selected Earth-centred–Earth-fixed (ECEF) reference frame (Ashby 2005). Combining equations (6) and (7) and dropping the obvious constant-frequency terms, we obtain

$$\begin{aligned} \tau_s = \int \frac{\Delta f}{f} dt = \int \left[-\frac{GM}{c^2 r} + \frac{GM J_2}{c^2 r} \left(\frac{a_1}{r} \right)^2 \right. \\ \left. \times \left(\frac{3z^2}{2r^2} - \frac{1}{2} \right) - \frac{1}{2} \frac{v^2}{c^2} \right] dt. \end{aligned} \quad (8)$$

This relativistic effect is computed using numerical integration, and additional constant-frequency terms are dropped. Because the constant-frequency terms have been dropped, τ_s has been redefined. Whereas before it represented the accumulated proper time kept by the satellite clock, it now represents only the time-varying part. This is, in fact, the end that was sought: we now have a more general expression for the periodic relativistic effect, and the τ_s of (8) can now be directly compared with the approximate effect given in (2).

The primary interest of this paper is the GRACE satellites. GRACE's eccentricity varies between 0.0015 and 0.0024 with a ~ 94 day period. Equation (1) thus predicts that the amplitude of the once/rev relativistic clock effect will also vary with a 94 day period (figure 1). In figure 2(a) numerical integration of equation (8) is compared with $-2(\mathbf{r} \cdot \mathbf{v})/c^2$ for high- and low-eccentricity GRACE orbits. As expected, the amplitude of the relativistic clock effect on the GRACE satellite varies significantly with eccentricity; the difference between using $-2(\mathbf{r} \cdot \mathbf{v})/c^2$ and numerical integration to compute the relativistic effect is similar. This is difficult to see when the relativistic effect is at its peak, so a close-up for the larger-eccentricity orbit is shown in figure 2(b).

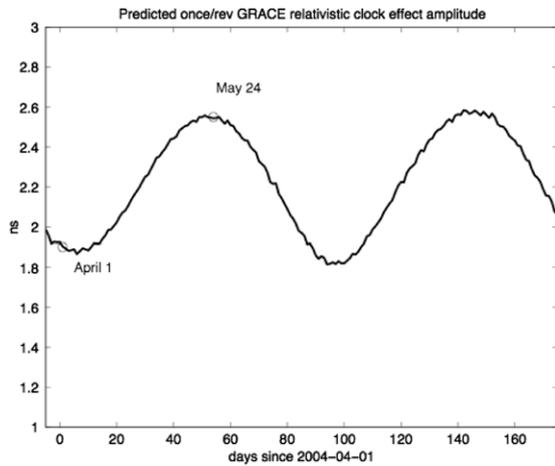


Figure 1. Amplitude of the once/rev relativistic clock effect predicted for the GRACE satellites using equation (1).

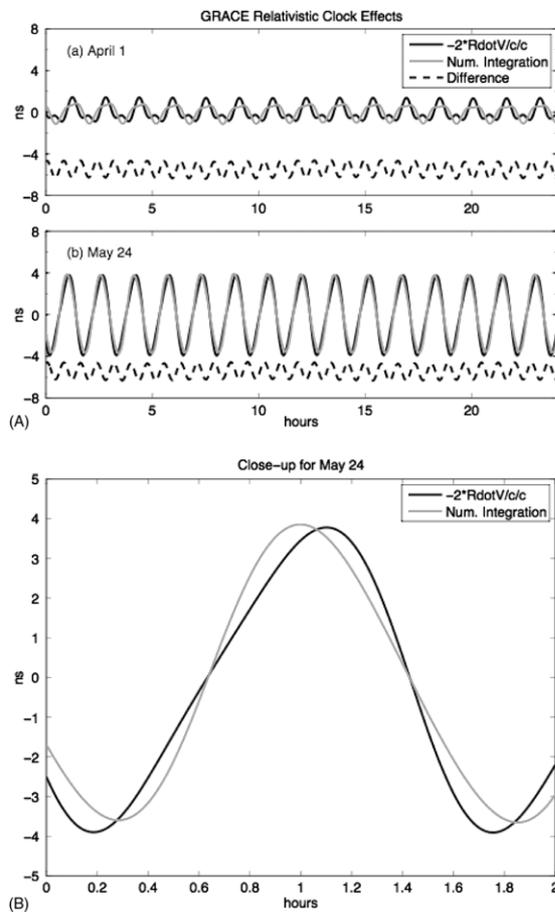


Figure 2. (A) GRACE relativistic clock effects computed using equations (2) and (8). The difference between the two calculations is shown offset below each set of series. GRACE A is shown. The data shown were collected in 2004. (B) Close-up view of relativistic clock effects for GRACE A on 24 May 2004.

We also examine relativistic effects for the TOPEX altimetry satellite. TOPEX’s eccentricity (~ 0.00013) is much smaller than GRACE’s; therefore, the relativistic correction which must be applied to TOPEX’s clock is also much smaller. Figure 3 shows a comparison of the effects obtained from

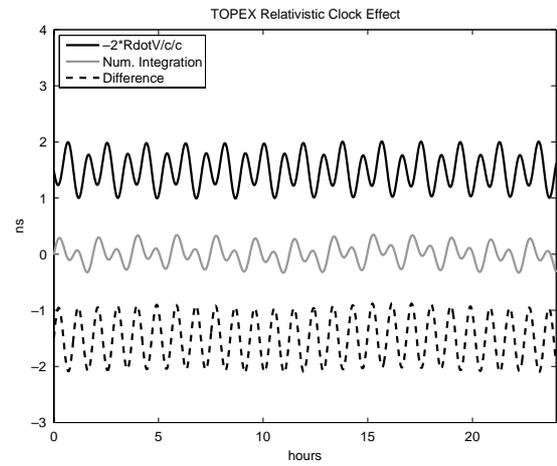


Figure 3. Relativistic clock effects for TOPEX using $-2(\mathbf{r} \cdot \mathbf{v})/c^2$, numerical integration of (8) and the difference between the two. These were computed for data from 5 May 2005 provided by the Center for Space Research (John Ries, personal communication). The three time series are offset for purposes of display.

numerical integration and using $-2(\mathbf{r} \cdot \mathbf{v})/c^2$. The effects obtained from the two methods differ markedly; in fact, the size of the relativistic effect is smaller than the error made by assuming that equation (2) is correct. In this case, one would be better off making no relativistic correction than applying $-2(\mathbf{r} \cdot \mathbf{v})/c^2$. The effects of J_2 for the GPS satellite clocks are much smaller, but still observable, as discussed by Kouba (2004).

The numerical integration performed in (8) requires that precise position and velocity data be available at sufficiently small time steps; we used 30 s intervals in computing results for the two LEO satellites. Since precise position and velocity data are frequently available for missions using dual-frequency GPS receivers, the numerical integration of equation (8) should be used.

3. GRACE data analysis

With GPS observations from two receivers, a user can estimate the time difference between the two oscillators being used to drive the GPS receivers. This timing estimate is fundamentally based on the satellite pseudorange measurements, which are several orders of magnitude noisier than the carrier-phase data. If both pseudorange and carrier-phase data are used with precise geodetic models, very precise and accurate ‘geodetic’ time and frequency transfer results have been shown for ground systems (Schildknecht *et al* 1990, Baeriswyl *et al* 1994, Larson *et al* 2000, Dach *et al* 2005, Ray and Senior 2003, Plumb and Larson 2005). Now that geodetic-quality GPS receivers are being flown in space, geodetic time-transfer techniques can be tested in this more challenging environment.

In order to examine the effects of relativity on an orbiting clock, we need an accurate transfer system (provided by geodetic-quality GPS) and an oscillator that is stable enough to observe the expected relativistic signals discussed in the previous section. There have been very few space missions that have flown both stable oscillators and geodetic-quality GPS receivers. The first GPS receiver/stable oscillator

combination was launched on TOPEX in 1992. Unfortunately, the evaluation of those data was compromised when the GPS receiver lost its L2 capability after anti-spoofing was activated in 1994. Ten years later, NASA launched the GRACE satellites. These satellites are both equipped with GPS receivers and free-running ultra-stable oscillators (USOs) (Wallis *et al* 2005). The receivers operate on both frequencies in the presence of anti-spoofing. Precise orbits for the GRACE satellites are routinely estimated, making it possible to test the relativistic effects derived in the previous section.

The GPS (BlackJack) receivers (Montenbruck and Kroes 2003) on the GRACE satellites are capable of simultaneously tracking up to 24 dual-frequency signals, i.e. 12 satellites. Once per second (1 Hz) GPS data from each satellite are transmitted to the ground, edited for cycle slips and outliers, and decimated to 5 min pseudorange and carrier-phase samples. By convention, the GPS ground network records observations at 0 and 30 s after the minute. Since the USOs used on the GRACE satellites are free-running, soon after launch their observations are no longer synchronized with the ground GPS network. A low-order polynomial is thus used to propagate the GRACE measurements to the measurement times used by the ground network. The individual L1 and L2 frequency data are combined to remove the largest effects of the ionosphere. In this study, the GIPSY software (Lichten and Border 1987) and the JPL FLINN GPS products (Jefferson *et al* 1999) were used to estimate both time and position of the GRACE spacecraft.

The basic principles of carrier phase (also known as geodetic) time transfer are outlined in Larson and Levine (1999). The positions of the transmitting GPS satellites are held fixed and the positions of the receivers are estimated in a terrestrial reference frame; in this study ITRF2000 was used (Altamimi *et al* 2002). Because the GPS receivers discussed in this study are on low Earth orbiters, both their positions and velocities at a given epoch are estimated rather than a single Cartesian position as is done for ground receivers. When the JPL FLINN products are used, the transmitter clocks are also fixed to values determined from a previous least-squares analysis of GPS carrier phase data from a global tracking network. These transmitter clocks themselves are estimated relative to a ground clock that is held fixed. For the FLINN analysis, the fixed (reference) clock is generally defined as the Alternate Master Clock (AMC2) at the GPS Master Control Station at Schriever Air Force Base (Colorado Springs, CO). The net result of this analysis is that all estimated clocks are defined relative to the AMC2, a hydrogen maser steered in the long-term to agree with the Master Clock at the United States Naval Observatory (USNO). The Master Clock at USNO is steered to Universal Coordinated Time (UTC) and provides USNO's realization of UTC, UTC(USNO). Additional information related to force models and specific GRACE processing strategies can be found in Bertiger *et al* (2002).

The largest relativistic corrections have periods of once/rev and twice/rev. To emphasize these periods, a fifth order polynomial has been estimated and removed from the GRACE clock estimates. The typical quality of GRACE clocks estimated for this study is shown in figure 4. In the bottom panel these same clock estimates are shown after

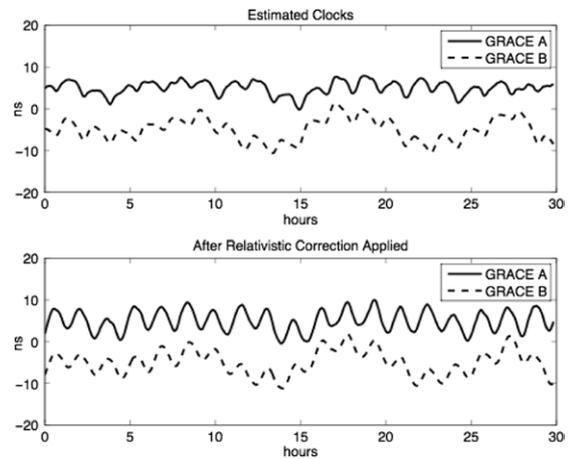


Figure 4. 21 November 2004 GRACE clock estimates: (top) with fifth order polynomial removed; (bottom) with fifth order polynomial and relativistic effects removed.

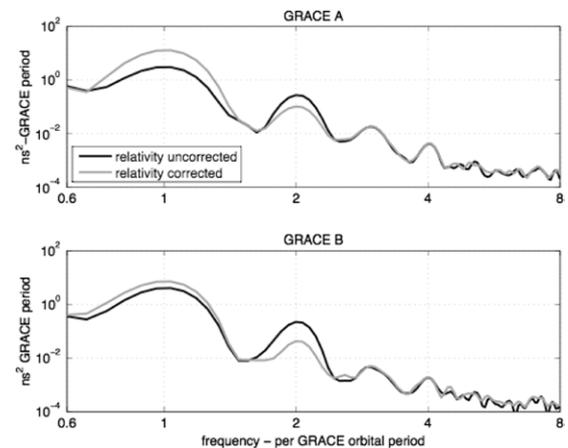


Figure 5. Power spectra for time series. Top: GRACE A; bottom: GRACE B.

relativistic effects have been removed. The effect of removing the relativistic contributions has not removed once/rev power; in fact it is now larger than it was before the measurements were corrected. This can be seen more clearly in figure 5, where Welch periodograms have been computed for the 30 h time series for both GRACE satellites. The power spectral peak at once/rev is larger after the relativistic correction; the twice/rev peak is significantly smaller. Also notable in figure 5 is that the once/rev power is larger for GRACE A than for GRACE B.

These data were further examined by evaluating clock estimates for GRACE A and GRACE B for a six-month period. Because the eccentricity of GRACE varies with a 94 day period, a long-term study allows us to examine whether the relativistic corrections are correctly removing the eccentricity dependence (figure 1). Since twice/rev power does not depend significantly on eccentricity, it should be reduced throughout the six-month period. Figure 6 shows once/rev power spectral amplitudes for GRACE A and GRACE B before and after relativistic corrections have been applied. As expected, once/rev power for each satellite no longer varies with eccentricity. This plot also confirms that the GRACE A clock has more once/rev power than the GRACE B clock. The

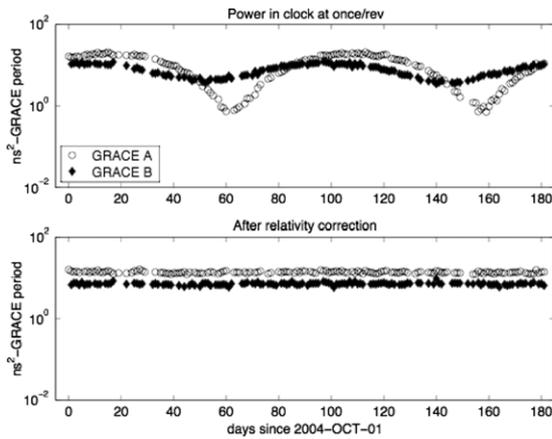


Figure 6. Once/rev power in GRACE clock estimates before (top) and after (bottom) relativistic effects have been removed.

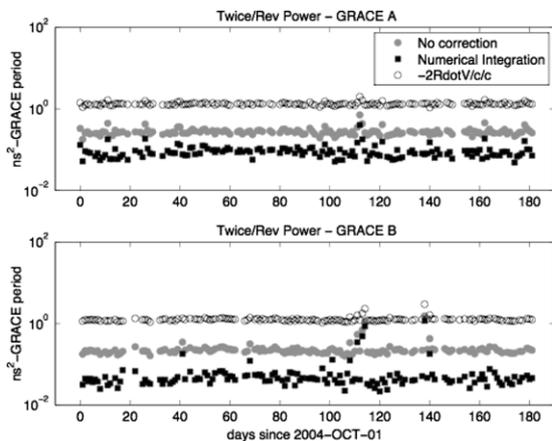


Figure 7. Twice/rev power for GRACE A (top) and GRACE B (bottom). Power has been calculated for three cases: no relativistic correction, numerical integration correction for relativity and $-2(\mathbf{r} \cdot \mathbf{v})/c^2$.

average amplitudes of the once/rev signals are 3.4 ns and 2.4 ns for GRACE A and GRACE B, respectively.

The twice/rev power spectral amplitudes are shown in figure 7. There is no significant dependence on eccentricity. For twice/rev power, using the incorrect relativistic expression $(-2(\mathbf{r} \cdot \mathbf{v})/c^2)$ significantly increases power rather than what is expected, a reduction. This is consistent with what was shown in the previous discussion: $-2(\mathbf{r} \cdot \mathbf{v})/c^2$ does not properly account for the effects of J_2 , and introduces clock errors with a frequency of twice/rev. Likewise, the numerical integration results significantly reduce the twice/rev amplitudes from 0.53 ns and 0.48 ns to 0.30 ns and 0.23 ns for GRACE A and GRACE B, respectively.

4. Noise sources and the GRACE clock estimates

We next examine possible sources for the remaining once/rev and twice/rev signals in the relativity-corrected GRACE clock estimates. One potential error source is the GPS time-transfer system. Any error in modeling the GRACE spacecraft environment or in models that were used to estimate transmitter clocks also contributes to the GPS time transfer system error

budget. Because the period of the largest remaining clock signal is once/rev, the error sources are likely related to the GRACE spacecraft and the instruments onboard (e.g. the USO or GPS receiver).

Three possibilities were examined for the once/rev clock signals: temperature, ionizing radiation and voltage, which we will discuss in turn. Temperature effects have frequently been seen in GPS time-transfer results (Overney *et al* 1997). Some of the errors in the GPS clock estimates are related to the GPS instrumentation rather than to the oscillator itself (Larson *et al* 2000). The GPS transfer system can be affected by temperature at the receiver, cable or the antenna. Temperature measurements collected inside the spacecraft confirm that the receiver did not sense significant temperature changes (0.1 °C) and thus is unlikely to have produced the observed once/rev GRACE signals. The GPS antenna does experience more than 4 °C temperature variations with a period of once/rev. These antenna temperatures also vary seasonally though, as the geometry with respect to the sun changes. Recall that the once/rev power in the GRACE clocks does not vary with time, and thus changes in antenna temperature are also an unlikely source for the once/rev signals. Since one of the GRACE spacecraft is flying ‘backwards’, it was also considered that there could be different thermal regimes due to spacecraft orientation. This hypothesis was tested by examining GRACE clock estimates before and after the satellites switched places in December 2005. The once/rev clock amplitudes on GRACE A and GRACE B did not change.

The observed once/rev clock signatures could be caused by the GRACE USOs themselves. Their thermal stability was measured prior to launch at better than 4 parts in $10^{13}/^{\circ}\text{C}$. Temperature measurements collected inside the spacecraft were examined: specifically from the USO, microwave assembly, sampler and instrument processing units. The microwave assembly (containing the amplifiers and analogue electronics) and USO measurements never vary more than 0.1 °C within 24 h. The variations on the sampler and instrument processing units are more significant (0.4 °C). Based on prelaunch thermal sensitivity measurements, none of these temperature variations would be large enough to cause the once/rev timing changes observed for GRACE A and GRACE B.

Quartz resonators such as the GRACE USOs are also known to be sensitive to ionizing radiation, roughly 3 parts in 10^{12} per South Atlantic Anomaly (SAA) pass. Most recently this effect was observed on the USO for the JASON spacecraft (Lemoine and Capdeville 2006). The strong dependence of the GRACE clock estimates on orbital period does not match what would be predicted for SAA (Matt Reinhart, personal communication), where one would expect variations only when the spacecraft crossed a particular region of the Earth.

The GRACE USOs have a small frequency/voltage sensitivity, typically less than 1 part in 10^{12} fractional frequency change per volt (Wallis *et al* 2005). Unlike the much smaller temperature variations within the spacecraft, there are large once/rev variations in the battery voltages for the GRACE satellites, with a peak-to-peak difference of 5 V (figure 8). Although the need for a voltage regulator upstream of the accelerometers was recognized, no testing was done for the USOs, and thus the plotted voltage variations are

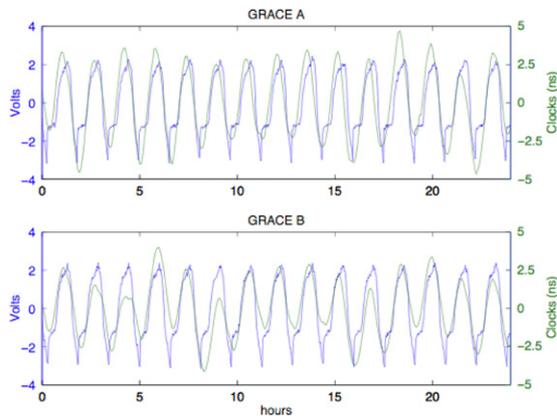


Figure 8. GRACE A (top) and GRACE B (bottom) voltage variations and clock estimates (means removed from both time series). Data for 13 December 2004 are shown.

(This figure is in colour only in the electronic version)

typical of the voltage variations that were input to the GRACE USOs. The voltage variations do not match the GRACE clocks exactly, but align reasonably well in time. One inconsistency in the hypothesis of voltage-induced clock variations is that the measured voltage variations between the two satellites are similar, while the once/rev GRACE clock amplitudes differ by 50%. This inconsistency can be resolved because individual USOs of this type can vary in voltage sensitivity by a factor of 2 (Matt Reinhart, personal communication, 2006). We therefore conclude that it is plausible that GRACE A and GRACE B once/rev clock signatures reflect a response to large once/rev voltage variations. For example, the quoted sensitivity of 1 part in 10^{12} for the voltage variations observed on GRACE could have produced ~ 27 ns peak-to-peak variations, much larger than the 5 ns to 7 ns variations that were actually observed. Although the observed once/rev GRACE USO signatures are large, they are not significant at the short periods (< 1 s) that are critical for the success of GRACE's scientific mission.

5. Conclusions

A relativistic expression that explicitly includes the effect of J_2 has been derived for Earth-orbiting satellites. GPS data from the GRACE satellites were used to show that subtracting the relativistic contributions removes eccentricity dependence in the once/rev signals and significantly reduces twice/rev clock signals, as is consistent with theory. While easier to implement than the numerical integration expression used in this paper, using the relativistic correction expression derived for GPS, $-2(\mathbf{r} \cdot \mathbf{v})/c^2$, produces significant and measurable errors for the two low Earth orbiters (TOPEX and GRACE) studied here. After correction for relativistic effects, a large once/rev signal remains in the GRACE clock estimates, with amplitudes of 3.4 ns and 2.4 ns for GRACE A and GRACE B, respectively. This does not appear to be linked to the thermal environment on the spacecraft. Voltage variations on the two spacecraft provide one plausible explanation for the large timing variations observed in the GRACE data.

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References

- Altamimi Z, Sillard P and Boucher C 2002 ITRF2000: a new release of the International Terrestrial Reference Frame for earth science applications *J. Geophys. Res.* **107** 2214 doi:10.1029/2001JB000561.
- Ashby N 2005 Relativity in the Global Positioning System *100 Years of Relativity* ed A Ashtekar (Singapore: World Scientific) ISBN 981-256-394-6; also see <http://relativity.livingreviews.org/Articles/lrr-2003-1/>
- Ashby N and Spilker J J 1996 Introduction to relativistic effects *Global Positioning System: Theory and Applications* vol 1 ed B Parkinson and J J Spilker Jr (Reston, VA: American Institute of Aeronautics and Astronautics) pp 623–95
- Baeriswyl P, Schildknecht Th, Utzinger J and Beutler G 1994 Frequency and time-transfer with geodetic GPS receivers: first results *Proc. 9th EFTF (Besançon, France)* pp 46–51
- Bertiger W *et al* 2002 GRACE: millimeters and microns in orbit *Proc. ION GPS (Portland, OR, September 2002)*
- Bertiger W, Dunn C, Harris I, Krusinga G, Romans L, Watkins M and Wu S 2003 Relative time and frequency alignment between two low Earth orbiters, GRACE *Proc. 2003 UFFC Conf. (Tampa, FL, May 2003)*
- Beutler G, Mueller I I and Neilan R E 1994 The International GPS Service for Geodynamics (IGS): development and start of official service on January 1, 1994 *Bull. Geodesique* **68** 39–70
- Dach R, Hugentobler U, Schildknecht T, Bernier L-G and Dudle G 2005 Precise continuous time and frequency transfer using GPS carrier phase *Proc. 2005 IEEE Int. Freq. Contr. Symp. (Vancouver, Canada)* vol 29–31, pp 329–36
- IAU 1991 Resolution, Recommendation IV on Terrestrial Time, available online at <http://www.iers.org/MainDisp.csl?pid=98-109>
- IAU 2000 Resolution adopted by the 24th General Assembly, available http://syrt.eospm.fr/IAU_resolutions/Resol-UAI.htm
- ICD-GPS-200c 1993 Interface Control Document, NAVSTAR GPS Space Segment, Navigation User Interface, October 10, Revision C, AIR Inc. Research Corporation (see also IS-GPS200 Revision D, 7 March 2006)
- Jefferson D, Bar-Sever Y, Heflin M, Watkins M, Webb F and Zumberge J 1999 JPL IGS Analysis Center Report, *IGS 1998 Technical Reports* pp 89–97
- Kouba J 2004 Improved relativistic transformations in GPS *GPS Solutions* **8** (3) 170–80
- Larson K M and Levine J 1999 Carrier phase time transfer *IEEE Trans. UFFC* **46** 1001–12
- Larson K, Levine J, Nelson L and Parker T 2000 Assessment of GPS carrier-phase stability for time-transfer applications *IEEE Trans. UFFC* **47** 484–94
- Lemoine J and Capdeville H 2006 A corrective model for Jason-1 DORIS Doppler data in relation to the South Atlantic Anomaly *J. Geod.* **80** 507–23 doi:10.1007/s00190-006-0068-2
- Lichten S M and Border J S 1987 Strategies for high-precision global positioning system orbit determination *J. Geophys. Res.* **92** 12 751–62
- Montenbruck O and Kroes R 2003 In-flight performance analysis of the CHAMP BlackJack GPS receiver *EGS—AGU—EUG*

- Joint Assembly (Nice, France, 6–11 April 2003)* abstract #4016
- Overney F, Schildknecht T, Beutler G, Prost L and Feller U 1997 GPS time transfer using geodetic receivers: middle-term stability and temperature dependence of the signal delays *Proc. 11th Eur. Frequency Time Forum (Neuchâtel, Switzerland)* pp 504–8
- Plumb J and Larson K 2005 Long-term comparisons between two way satellite time and frequency transfer and geodetic time transfer systems with TWSTFT *IEEE Trans. UFFC* **52** 1912–18
- Ray J and Senior K 2003 IGS/BIPM pilot project: GPS carrier phase for time/frequency transfer and time scale formation *Metrologia* **40** S270–88
- Schildknecht T, Beutler G, Gurtner W and Rothacher M 1990 Towards subnanosecond GPS time transfer using geodetic processing technique *Proc. 4th Eur. Frequency Time Forum (Besançon, France)* pp 334–56
- Wallis R E, Weaver G L, Reinhart M J and Cheng S 2005 An advanced synthesized ultra-stable oscillator for spacecraft applications *Proc. IEEE Aerospace Conf. (Big Sky, MT, March 2005)*