Geodetic measurements in Greenland and their implications

John Wahr,1 Tonie van Dam,2 Kristine Larson,3 and Olivier Francis2

Abstract. We describe results from an ongoing experiment in Greenland, in which we are using absolute gravity and continuous Global Positioning System (GPS) measurements to study vertical crustal motion at two locations along the edge of the ice sheet: Kellyville, located about one third of the way up the western ice margin, and Kulusuk, located along the eastern ice margin at about the same latitude as Kellyville. The GPS measurements suggest average crustal uplift rates of $-5.8 \pm 1.0$ mm/yr at Kellyville and $-2.1 \pm 1.5$ mm/yr at Kulusuk. There have not yet been enough absolute gravity occupations to permit useful secular gravity solutions at either location. The negative uplift rate at Kellyville is consistent with independent archeological and historical evidence that the southwestern edge of the continent has been subsiding over the last 3000 years, but it is inconsistent with estimates of the Earth’s continuing viscoelastic response to melting ice during the early Holocene, which predict that Kellyville is likely to be uplifting, rather than subsiding, by $2.0 \pm 3.5$ mm/yr. The resulting $-7.8 \pm 3.6$ mm/yr discrepancy between the observed and predicted uplift rates is too large to be caused by loading from present-day changes in nearby ice. However, it is consistent with independent suggestions that the western ice sheet margin in this region may have advanced by $\approx 50$ km during the past 3000-4000 years. If this advance did occur and if the crustal subsidence it induces is not removed from altimeter measurements of Greenland ice sheet elevations, then the altimeter solutions could underestimate the true snow/ice thickness change by 5-10 mm/yr along portions of the western margin of the ice sheet.

1. Introduction

Estimates of the present-day mass imbalance of the Greenland ice sheet are highly uncertain. Warrick et al. [1996] concluded that Greenland’s contribution to global sea level rise over the last century could have been anywhere from $-0.4$ to $+0.4$ mm/yr, corresponding to a rate of ice thickness change averaged over the entire ice sheet of between $+85$ and $-85$ mm/yr. Measurements from satellite radar altimeters [Zwally et al., 1989; Davis et al., 2000] and airborne laser altimeters [Krabill et al., 1999] have led to estimates of the mass imbalance over portions of the ice sheet. In the future, NASA’s ICE-Sat laser altimeter satellite [Schutz, 1998] and the European Space Agency (ESA) CRYOSAT radar altimeter mission [Wingham, 1999] will greatly increase this coverage.

Altimeter observations, however, detect changes only in ice sheet elevations. Inferring mass imbalance from those elevations requires knowledge of the density profile throughout the upper layers of the snow column. This density profile is constantly changing because of variability in accumulation rate. The uncertainty in the density profile is one of the major error sources for altimeter estimates of mass imbalance [see, e.g., Arthern and Wingham, 1998].

Wahr et al. [1995] describe a method of combining GPS and absolute gravity measurements made on the Earth’s crust adjacent to an ice sheet to constrain changes in nearby ice mass. The method is directly sensitive to mass variability and does not require knowledge of the snow density profile. This approach has limited spatial resolution and should best be viewed as a means of validating altimeter mass imbalance estimates averaged over regions of the ice sheet.

Let $u$ be the vertical uplift of the crust caused by a combination of the Earth’s elastic response to present-day changes in ice load and its viscoelastic response to ice variations in the past. Let $\delta g$ be the perturbation in the gravitational acceleration at the Earth’s surface due to this same combination of effects but after being corrected for the vertical motion of the instrument caused
by the motion of the crust on which it sits. Wahr et al. [1995] found that the quantity
\[ \Delta = u - (6.5 \text{ mm}/\mu\text{Gal}) \delta g \] (1)
depends almost entirely on the Earth's elastic response to present-day changes in ice mass and is nearly inde-
pendent of the viscoelastic response to past ice variability.

We have initiated a limited observational campaign in Greenland to implement this technique. We installed continuously tracking GPS receivers at two bedrock locations on opposite sides of the ice sheet (Figure 1). One receiver (a Turborogue SNR-8000, with a chokering antenna and a radome), was installed in July 1995 along the western edge of the ice sheet at Kellyville (66.9873°N, 309.0552°E), the location of the Sondres-
strom Radar Facility near Kangerlussuaq. The facility is funded by the NSF Upper Atmosphere Facilities Pro-
gram and operated by SRI International. The other receiver (an eight-channel Trimble 4000 SSI) was installed in July 1996 near the Kulusuk airport (65.5793°N, 322.8506°E), along the eastern edge of the ice sheet and 635 km distant from Kellyville. Initially, a Trim-
ble ground plane antenna with a hemispherical radome was used at Kulusuk. It was replaced with a chokering antenna and a SCIGN radome on May 10, 1999.

Both the Kellyville and Kulusuk sites are on bedrock. The Kellyville antenna is attached to the bedrock with an invar rod, which has only a small coefficient of thermal expansion. The monument at Kulusuk is a pre-
existing concrete pillar that was used by the Danish National Survey and Cadastre (KMS) for many years to make stellar triangulation measurements. The Kel-
lyville data are archived both at National Oceanic and Atmospheric Administration (NOAA) and at the In-
ternational GPS Service (IGS). The Kulusuk data are archived at the University NAVSTAR Consortium (UN-
AVCO).

We have visited each site with an NSF-owned FG5 absolute gravity meter [Bilham and Sasagawa, 1994] for 1-
2 weeks every summer, beginning in 1995 at Kellyville and 1996 at Kulusuk. When not in use, the meter is main-
tained by NOAA’s Table Mountain Gravity Observatorv in Boulder. The gravimeter observations are made on bedrock, within 10-20 m of the GPS sites at each location. There is no evidence of local faulting at either site, implying that the GPS antenna and the gravimeter should experience the same vertical motion.

**Figure 1.** Our predicted crustal uplift rates (in mm/yr) caused by postglacial rebound, computed by convolving our viscoelastic Green’s functions with three models of the late Pleistocene/early Holocene ice history: (a) the Greenland component of ICE-3G [it Tush-
ingham and Peltier, 1991], (b) the Greenland simulation of Le Meur and Huybrechts [1998] (referred to as HUY2 in the text), not including any changes in ice during the last 4000 yr, and (c) all ICE-3G ice variability outside Greenland. The locations of the GPS sites at Kellyville and Kulusuk are denoted with a plus sign and a triangle, respectively. The Green’s functions are computed using the default viscosity profile and the elastic parameters described in the text.
We show below that there are not yet enough gravity data to determine $\Delta$ well enough for it to be useful. But the GPS measurements by themselves are providing interesting and unanticipated results. Especially notable is a secular uplift rate of $-5.8 \pm 1.0$ mm/yr that we infer below from the GPS data at Kellyville (a negative uplift rate implies subsidence). We will show that this negative uplift rate conflicts with estimates of the Earth's viscoelastic response to changes in ice mass during the late Pleistocene and early Holocene. Furthermore, the magnitude of the observed Kellyville rate is far too large to be caused by the Earth's elastic response to any plausible, ongoing change in nearby ice mass. Our tentative hypothesis will be that this subsidence could reflect the Earth's continuing viscoelastic response to a previously proposed readvance of the western ice sheet margin that may have begun 3000-4000 years ago.

2. Observations

2.1. GPS Data

We have analyzed the GPS observations at Kellyville and Kulusuk using the GIPSY/OASIS II software developed at the Jet Propulsion Laboratory (JPL) [Zumberge et al., 1997]. After data editing, the carrier phase and pseudorange observations from both GPS frequencies are linearly combined to remove nearly all ionospheric effects. We use GPS orbits, Earth orientation, and clock products produced by JPL's independent analysis of ~40 globally distributed, continuously operating GPS receivers. The remaining parameters to be estimated are station position, tropospheric refraction, and receiver clock parameters.

Figure 2 shows daily values of vertical positions at Kellyville and Kulusuk. The $1\sigma$ formal uncertainties (not shown) for the daily vertical values are of the order of 15-20 mm. For Kellyville there was a 3-4 month period of data outage and sporadic returns during the winter of 1995-1996 that we believe was caused by radio interference. The problem was fixed by upgrading the GPS antenna. Since then, the data recovery has been more or less continuous. Note that a secular decrease in the vertical is clearly visible across the 5 year Kellyville record.

The Kulusuk record contains several large data gaps caused by various hardware and software problems. In
addition, there are clear nonsecular excursions in the Kulusuk data, including a sharp subsidence and recovery in the latter part of 1998 (identified with the arrow) for which we have no explanation. This offset has been reproduced by independent analysis (T. Herring, personal communication, 2000). Quality flags in the data lead us to believe the receiver was not functioning properly during this period.

2.2. Gravity Data

The absolute gravity results are shown in Figure 3. Each point represents 72 hours to 2 weeks of observations, depending mostly on flight schedules between the United States and Greenland and between Kellyville and Kulusuk. The data are processed using the package provided by the manufacturer, Micro-G. Atmospheric pressure effects are removed using an admittance factor of $-0.3 \, \mu\text{Gal/mbar}$. The effects of polar motion are removed using International Earth Rotation Service (IERS) pole positions. All other instrumental corrections, including those for relativity, for the height of the drop, for the gravity gradient, etc., have been applied. At Kulusuk, Earth tides have been removed using an Earth tide model, and ocean loading tides are computed using the Schwiderski ocean tidal model [Schwiderski, 1980]. Earth and ocean loading tides have been removed from the Kellyville data using observed tidal parameters [Scheinert et al., 1998]. All gravity data are archived at NOAA in Boulder.

The vertical lines in Figure 3 represent our best estimates of the $1\sigma$ uncertainties and are based on the observed scatter in the data. The uncertainties vary from year to year and depend largely on experimental conditions. The large offset in the Kulusuk data between the 1997 and 1998 occupations is largely due to the construction of a hotel in the intervening months, which comes to within a few meters of the site and is somewhat downhill from it.

Converted to errors in equivalent vertical crustal motion, the single-point uncertainties shown in Figure 3 agree favorably with those from GPS. However, because our GPS results are continuous throughout the year, the contributions from random errors and other nonsecular terms will average out more quickly in the GPS solutions than in the gravity results. We will show in section 6 that the trends inferred from the gravity data are not yet well enough determined for them to be useful for estimating $\Delta$ (equation (1)).

2.3. Fitting to the GPS Data

Here we illustrate our method of determining secular trends in the GPS data by considering the data from Kellyville. We first least squares fit a secular term and the sine and cosine of an annual term to the daily data. Initially, we weight each data point using the $1\sigma$ formal errors generated by the Gipsy/OASIS II software during the GPS analysis and assume those errors are uncorrelated from one day to the next.

Figure 4a shows the daily Kellyville data, along with the sum of the best fitting secular and annually varying terms (the solid line). The daily residuals (the differ-
ences between the daily values and the solid line) are clearly not entirely random. They arise from a combination of measurement errors, modeling errors (including tropospheric and orbit errors), and real vertical motion that cannot be described with secular or seasonal terms. Our initial fitting process, which assumed the daily residuals are uncorrelated with one another, is thus inconsistent with these residuals and is likely to underestimate the uncertainties in the fitted parameter values.

To overcome this problem, we construct multiday averages of the GPS data. We estimate the decorrelation time for the residuals by taking the autocorrelation of the daily residuals

\[ C(\tau) = \frac{\sum t h(t) h(t - \tau)}{\sum t h(t)^2}, \]  

Figure 4. Figure 4a shows the daily GPS values for the de-meaned vertical position at Kellyville. The solid line is the sum of the best fitting trend and annually varying terms. Figure 4b shows the autocorrelation of the GPS Kellyville residuals (the differences between the daily values and the solid line shown in Figure 4a). Figure 4c shows the 41-day averages of the GPS vertical measurements at Kellyville. The solid line is the sum of the best fitting secular and annually varying terms.
where \( h(t) \) is the residual at time \( t \). The autocorrelation results for Kellyville are shown in Figure 4b. If the daily residuals were uncorrelated, the autocorrelation would be 1 for no lag (i.e., for \( \tau = 0 \) in (2)), but for all \( \tau > 0 \) it would be normally distributed about zero with variance \( 1/N \), where \( N \) is the number of residuals [see, e.g., Jenkins and Watts, 1968].

Note from Figure 4b that the autocorrelation at Kellyville does decrease notably from \( \tau = 0 \) days (where \( C(0) = 1.0 \) by definition) to \( \tau = 1 \) days (where \( C(1) = 0.27 \)). This implies there is a significant random component to the residuals. But the results also show that the autocorrelation does not decay to a normally distributed function about zero until the lag time approaches 65-70 days. The implication is that there is a nonnegligible component of the residuals that is correlated over about a 2 month period. On the other hand, we note that the magnitude of the autocorrelation has decreased to its \( \tau = \infty \) limit at lag times of \(~40 \) days and longer.

As a compromise between choosing long averaging times to minimize the effects of correlated errors and choosing short averaging times to take advantage of the fact that much of the error in the daily observations is, indeed, random, we choose to construct 41-day averages. We simultaneously fit secular and annually-varying terms to these averages. Figure 4c shows the 41-day Kellyville averages and the best-fitting secular trend plus annually varying solution (solid line).

Our fit results, and especially our error estimates for these results, depend on the errors we assign to each 41-day average. We assign them by removing the best fitting secular and annually varying terms from the 41-day averages to construct 41-day residuals. Those residuals have an RMS of 4.8 mm, and so we assign an error of 4.8 mm to each 41-day average during the fitting process. The resulting uplift rate at Kellyville is \(-5.8 \pm 0.4 \) mm/yr.

However, this error estimate does not include the possible effects of a secular reference frame drift, which would be observationally indistinguishable from true secular motion of a station. To account for these effects, we follow Argus and Hefton [1995] and add (in quadrature) an uncertainty of \( \pm 5/(\text{number of years of data}) \) mm/yr (= \( \pm 0.9 \) mm/yr for Kellyville and \( \pm 1.1 \) mm/yr for Kulusuk) to the errors for the secular uplift rate obtained with the procedure described above. Our final, best estimate of the secular uplift at Kellyville is \(-5.8 \pm 1.0 \) mm/yr.

As a check on the robustness of our secular solution, we repeat this procedure for subsets of the Kellyville data of varying lengths: using the first 300 days of data, then adding the next 41 days, then the next 41 days, etc. Figure 5a shows the results for the secular trend plotted at the termination time of the data used in the inversion. The vertical line centered on each value represents the uncertainty from the fit, computed as described above except not including the reference frame uncertainty. Note that the value of the trend appears to have stabilized reasonably well by the beginning of 2000.

Figure 5b is similar but shows the fit to the daily values using the formal 1σ errors on those values. Although these are not our preferred secular trend estimates, the results do provide a consistency check. Here, again, the trend solutions look to have converged by the beginning of 2000.

For Kulusuk the method of analysis is similar. The autocorrelation of the Kulusuk data cannot be estimated as easily because of the significant gaps in the record, but the results (not shown) tentatively suggest that an averaging time of between 30 and 50 days is appropriate. We choose to construct 33-day averages of the Kulusuk data, and we repeat the fit procedure described above for Kellyville. We do not include data recorded during the time interval 1998.58-1998.72 when fitting to the Kulusuk data because of the unexplained subsidence and subsequent recovery that occurred during that time period (see the arrow in the bottom panel of Figure 2). Because of data gaps, some of the averages at Kulusuk are constructed from substantially fewer than 33 data points. Using averages over 33 days instead of over a longer time period tends to more evenly distribute the number of daily observations among the different average values.

Using these 33-day averages at Kulusuk, along with the fitting method described above for Kellyville, we obtain a secular uplift rate at Kulusuk of \(-2.1 \pm 0.7 \) mm/yr when the reference frame uncertainty is not included and \(-2.1 \pm 1.5 \) mm/yr after including the reference frame uncertainty. We adopt the latter as our best estimate of the uplift at Kulusuk. This \( \pm 1.5 \) mm/yr uncertainty also includes an estimated uncertainty due to the May 10, 1999, antenna replacement at Kulusuk. We assumed that this replacement caused a 0 \( \pm 2 \) mm shift in the data at the time of replacement [Schupler, 2000], and we went through our fitting process again after artificially shifting the data by 2 mm after May 10, 1999. We found a change of 0.7 mm/yr in the best fitting trend, which we interpret as an additional uncertainty in our original solution. We thus add \( \pm 0.7 \) mm/yr in quadrature to the other uncertainties in the solution. This increased the final uncertainty by \( \pm 0.2 \) mm/yr (the uncertainty was \( \pm 1.3 \) mm/yr before adding this additional uncertainty). Our final \(-2.1 \pm 1.5 \) mm/yr trend is not notably different from zero.

Figures 5c and 5d show the convergence of the 33-day and daily secular solutions at Kulusuk (the reference frame and antenna replacement uncertainties are not included in the error bars shown in these plots). Note that these solutions appear to have converged within our estimated \( \pm 1.5 \) mm/yr uncertainty, though the best fitting subsidence rate did increase slightly over the last 3-4 months of data.
3. Postglacial Rebound: Contributions From Ice Variability Prior to 4000 Years Ago

Tectonic motion (plate motion associated with mantle convection) is unlikely to be causing significant vertical motion at either Kellyville or Kulusuk. These sites are far from any active plate boundary or volcanic activity. For comparison, a recent study by Gurnis et al. [2000] concluded that southern Africa, a region similarly removed from plate boundaries but one that, unlike Greenland, may actually be actively uplifting, has experienced average uplift rates of 5-30 m/Myr over the last several tens of millions of years, corresponding to
only 0.005-0.03 mm/yr: >2 orders of magnitude smaller than the observed Kellyville rate.

Instead, any measurable secular vertical motion at either Kellyville or Kulusuk is presumably caused by a combination of the Earth's elastic response to present-day changes in ice and its continuing viscoelastic response to whatever changes in ice load might have occurred in the past. These two effects can be separated by combining the GPS uplift rate and the secular rate in gravity, but there have not yet been enough gravimeter occupations of either site to make this feasible. Until then, all we can do is to assess whether our GPS results are consistent with our expectations for the sum of the postglacial rebound (PGR) and present-day secular loading signals.

Here we discuss the likely effects of PGR on the uplift rates. A model of the rebound requires estimates of the Earth's viscosity structure and of the ice sheet glaciation-deglaciation history. Although changes in the Greenland ice sheet are obviously of greatest importance, the effects of the deglaciation of Canada and Scandinavia could also be significant.

The ICE-3G deglaciation model of Tushingham and Peltier [1991] describes changes in the global distribution of ice that occurred between 18,000 and 4000 years ago. ICE-3G includes deglaciation histories for Greenland, Canada, Scandinavia, and Antarctica as well as for some smaller regions (including Iceland). We use ICE-3G to represent all changes in ice outside of Greenland, after adding a 90 kyr linearly increasing glaciation phase to grow the ice sheets to their ICE-3G starting values.

To describe Greenland deglaciation prior to 4000 years ago, we consider three models. One is the Greenland component of ICE-3G, with the addition of the 90 kyr glaciation phase. (The present-day ICE-3G thickness values are used to begin the glaciation phase.) The others are results from two dynamical simulations: Huybrechts [1994] and Le Muer and Huybrechts [1998], which we refer to as HUY1 and HUY2, respectively. These simulations involve numerical modeling of the ice sheet's response to prescribed spatial and temporal distributions of surface mass balance and surface temperature. By comparing our results for ICE-3G, HUY1, and HUY2 we obtain estimates of the uncertainties in our predicted uplift rates caused by uncertainties in the Greenland ice model.

HUY2 is forced with a different temperature history than is HUY1 and uses a more rigorous method of incorporating crustal deformation, but otherwise, these two simulations are similar in their construction. One difference in the predictions of HUY1 and HUY2 which will be pertinent to the discussion in section 5, is that HUY1 shows a readvance of the ice sheet margin near Kellyville of up to several tens of kilometers during the last 3000-4000 years, whereas the readvance in HUY2 is much less pronounced. This is not an issue for the computations described in this section because here we do not include any HUY1 or HUY2 ice variability subsequent to 4000 years ago. This permits a more direct comparison with the results of ICE-3G. The possible effects of ice variability that might have occurred during the past 4000 years will be discussed in section 5. For both HUY1 and HUY2 we include a 90 kyr glaciation phase to grow the ice sheet to its initial value.

We convolve each ice model with viscoelastic Green's functions that describe the Earth's response to surface loading and that we compute using a collocation technique [see, e.g., Mitrovica and Peltier, 1992]. We keep spherical harmonic terms up to degree and order 70 in these calculations. The Green's functions are computed for an Earth with a four-layer viscosity profile, consisting of an elastic lithosphere overlying a viscous upper mantle/transition zone (the region between the bottom of the lithosphere and the seismic discontinuity at 670 km depth and denoted here with the subscript UM), a viscous lower mantle (the region between the 670 km discontinuity and the core-mantle boundary and denoted as LM), and an inviscid core. The upper and lower mantle viscosities are chosen to be uniform. Our default parameter values are 120 km for the lithospheric thickness and \( \eta_{UM} = 1 \times 10^{21} \text{ Pa s} \) and \( \eta_{LM} = 10 \times 10^{21} \text{ Pa s} \) for the upper and lower mantle viscosities.

For the elastic structure we assume the Earth is composed of five incompressible, homogeneous layers, with densities and shear wave velocities as given in Table 1. (Note that the elastic layers labeled as "transition zone" and "upper mantle" combine to form the
Table 2. Predicted Postglacial Rebound (PGR) Uplift Rates at Kellyville*

<table>
<thead>
<tr>
<th>Viscosity Model</th>
<th>Uplift Rate, mm/yr</th>
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<tr>
<td></td>
<td>ICE-3G</td>
</tr>
<tr>
<td></td>
<td>All Ice Components</td>
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<tr>
<td>Default model</td>
<td>3.6</td>
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<tr>
<td>$\eta_L M = 50 \times 10^{21}$ Pa s</td>
<td>3.1</td>
</tr>
<tr>
<td>$\eta_L M = 4.5 \times 10^{21}$ Pa s</td>
<td>3.5</td>
</tr>
<tr>
<td>$\eta_U M = 0.6 \times 10^{21}$ Pa s</td>
<td>2.6</td>
</tr>
<tr>
<td>Lithospheric thickness of 80 km</td>
<td>5.2</td>
</tr>
<tr>
<td>Lithospheric thickness of 210 km</td>
<td>1.4</td>
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* Uplift rates are computed by convolving our viscoelastic Green’s functions with the Greenland plus non-Greenland ice components of the ICE-3G model [Tushingham and Peltier, 1991], and with Greenland simulations from Huybrechts [1994] (HUY1) and from Le Meur and Huybrechts [1998] (HUY2). The contributions from the non-Greenland ice components of ICE-3G have been added to the HUY1 and HUY2 results. The Green’s functions are computed for the default parameters described in the text, unless noted otherwise in the Viscosity Model column.

UM viscous layer). By adopting this simplified elastic structure we are able to compute uplift rates for a wide range of viscosity profiles and for a large number of spherical harmonic degrees, with a minimum of effort. For certain viscosity profiles we have been able to compare our uplift rates with those computed by Han and Wahr [1995] for a compressible Earth with Preliminary reference Earth model (PREM) elastic values [Dziewonski and Anderson, 1981]. We have found agreement to within 20% or better. This level of discrepancy is smaller than the differences between uplift rates computed using different viscosity profiles and different Greenland ice models. We conclude that the use of our simplified elastic model will not notably affect our conclusions about the magnitudes of the PGR corrections and their uncertainties.

Figure 1 shows the predicted present-day uplift rates for Greenland and the surrounding region computed for the default viscosity model and driven by the Greenland portion of ICE-3G (Figure 1a), by the HUY2 simulation of Greenland ice variability prior to 4000 years ago (Figure 1b), and by all ICE-3G components outside of Greenland (Figure 1c). The locations of the GPS sites are indicated. Note that both HUY2 and ICE-3G predict that changes in Greenland ice prior to 4000 years ago contribute positive uplift rates at the two sites. This reflects the fact that after a mass load is removed, the Earth’s surface beneath the load continues rising to fill in the depression originally occupied by that mass. As the surface beneath the load uplifts, material is drawn in from surrounding regions. Thus the Earth’s surface out beyond the periphery of the load is likely to be subsiding. This explains the ≈2 mm/yr subsidence rate across Greenland evident in Figure 1c, caused mostly by the deglaciation of the ice sheets over present-day Canada.

The uplift rates predicted using HUY2 are notably smaller than those predicted using the Greenland component of ICE-3G. In fact, the HUY2 uplift rates at Kellyville and Kulusuk are not much larger than the subsidence rates predicted using the non-Greenland components of ICE-3G.

Our predicted uplift rates at Kellyville and Kulusuk are listed in Tables 2 and 3, respectively, for the default viscosity model and for variations of that model obtained by changing the lithospheric thickness and the

Table 3. Predicted PGR Uplift Rates at Kulusuk

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<td>1.7</td>
</tr>
<tr>
<td>$\eta_L M = 4.5 \times 10^{21}$ Pa s</td>
<td>1.3</td>
</tr>
<tr>
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</tr>
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upper and lower mantle viscosities to alternative but plausible values. Each result shown in Tables 2 and 3 is the sum of a Greenland contribution and a contribution from the non-Greenland component of ICE-3G. The predicted values are between −1.4 and 5.2 mm/yr at Kellyville and between −1.0 and 1.9 mm/yr at Kulusuk. Thus the PGR and GPS rates at Kulusuk are reasonably consistent with one another, but the rates at Kellyville are not.

To facilitate the removal of the PGR values from the GPS uplift rates, we interpret the numbers in Tables 2 and 3 as implying that the PGR uplift rates are in the range 2.0 ± 3.5 mm/yr at Kellyville and 0.5 ± 1.5 mm/yr at Kulusuk, and we interpret these uncertainties as 1σ Gaussian errors. This interpretation attaches special significance to the 2.0 and 0.5 mm/yr Kellyville and Kulusuk values, which perhaps is unwarranted. The 1σ error assumption implies there is a ≈32% probability that the correct PGR values are outside these intervals, and so these errors provide a reasonably conservative uncertainty estimate.

Removing the PGR rates from the GPS uplift rates and adding the PGR and GPS uncertainties in quadrature give residual uplift rates of $-7.8 \pm 3.6$ mm/yr at Kellyville and $-2.6 \pm 2.1$ mm/yr at Kulusuk. Thus the Kulusuk uplift rate is still not notably different from zero after removing the predicted PGR contribution. However, the unexplained (negative) uplift rate at Kellyville is not only still significant but has become larger.

Although we cannot be certain that the three Greenland ice models considered here are truly representative of all possible Greenland ice scenarios, it is unlikely that the real ice variation prior to 4000 years ago would be causing the present-day subsidence seen in the Kellyville GPS record. If it were, it would likely also have caused crustal subsidence prior to 4000 years ago, which would violate other PGR-related observations. For example, geological dating of Holocene shorelines along the west coast of Greenland, including in the vicinity of Kellyville, shows that the crust there was rising at an average rate of roughly 20 mm/yr between ∼9000 and 4000 years ago [Weidick, 1993], the time of the Greenland deglaciation as represented in ICE-3G. (The geological observations do not provide information about vertical crustal motion in this region for time periods after ∼4000 years before present for reasons that will be discussed in section 5.) In fact, even the relatively large ICE-3G predictions of uplift rates near Kellyville between 9000 and 4000 years ago are considerably smaller than those inferred from the geological observations (see the comparison with Sondre Stromfjord data in Figure 16 of Tushingham and Peltier [1992]).

4. Effects of Present-Day Ice Variability

A portion of the $-7.8 \pm 3.6$ mm/yr of unexplained Kellyville uplift rate could be due to the Earth’s elastic response to ongoing changes in nearby ice. To estimate the possible amplitude of this elastic motion, consider a half disc of ice (density of 917 kg/m$^3$) of radius $R$ that is changing its thickness at a rate of 750 mm/yr. Let the point P, representing Kellyville, lie just outside the straight edge of this half disc and at its center (see Figure 6 (top)). We use Farrell’s [1972] elastic Green’s function for vertical displacements to compute the subsidence rate at P. (Note viscous effects do not build up to be a significant fraction of the elastic effects until the load has been in place for several centuries [see Wahr and Han, 1998, Figure 1]). The subsidence rate is shown in Figure 6 (bottom) as a function of $R$. For a radius of $R = 600$ km the half disc would extend across the ice sheet to its eastern margin, so that this is effectively the
upper bound for \( R \). The results show that subsidence rates of 5-10 mm/yr, which is of the order of the residual subsidence rate at Kellyville, would be obtained for a disc radius of between 200 and 600 km. However, a present-day thickening rate of 750 mm/yr in the vicinity of Kellyville is 1-2 orders of magnitude larger than that inferred both from altimeter observations [Krabill et al., 1999; Davis et al., 2000] and from repeat GPS measurements on the ice sheet [Thomas et al., 2000]. Since the subsidence rates shown in Figure 6 scale linearly with the disc’s thickening rate, we conclude that the uplift rate caused by present-day changes in the ice sheet is probably 1-2 orders of magnitude smaller than the observed Kellyville uplift rate.

5. Postglacial Rebound Revisited: Contributions From Recent Ice Variability

Our GPS results are not the only observations that indicate recent or ongoing subsidence in this region. Archeological and historical evidence, including the apparent submergence of ancient Neo-Eskimo and Norse settlements along with more recent written records, suggest that the entire southern third of the west Greenland coast may have been subsiding over the last 3000 years at average rates from 3 mm/yr to up to as much as 10-20 mm/yr at some locations [Weidick, 1993, 1996]. These subsidence rates are imperfectly determined, because of difficulties in quantifying the measurements, but they are consistent with our present-day GPS Kellyville uplift rate of \(-5.8 \pm 1.0\) mm/yr.

The fact that all relevant measurements imply recent subsidence in this region, whereas the geological data imply that the crust was uplifting prior to 4000 years ago, suggests that the mode of the west Greenland ice sheet may have changed from one of contraction to one of expansion at some time between 3000 and 4000 years ago. The increasing load owing to the expansion would cause local subsidence owing to a combination of elastic and viscoelastic effects. This subsidence would presumably have caused the resubmergence of recently emerged crust, which would explain the paucity of geological evidence of relative sea level change in this region subsequent to 4000 years ago.

A recently advancing ice sheet margin near Kellyville is not included in the ICE-3G deglaciation model for Greenland, which in fact, makes no attempt to describe any changes in ice more recently than 4000 years ago. The HUY1 and HUY2 Greenland ice simulations do extend to the present day. HUY2 does not predict a significant readvance, but HUY1 does [see van Tatenhove et al., 1995]. The HUY1 model predicts that the ice sheet in this region retreated back through its present position \( \approx 8000 \) years ago and continued to retreat further back an additional \( \approx 50 \) km east of its present location, reaching a position of maximum retreat between 4000 and 5000 years ago. Between 3000 and 4000 years ago the HUY1 ice sheet began readvancing over that \( \approx 50 \) km to reach its present position sometime during the last few hundred years. The model does not show substantial thickening rates for ice in the interior during this readvance.

This general time evolution of the ice sheet margin is in good agreement with geological dating of moraines in the region, although the model timing tends to precede the geologically inferred timing by up to 1000 years, particularly during the recession stage [van Tatenhove et al., 1995]. The advance may also have been episodic, especially over the last few hundred years where periods of advance may have alternated with periods of recession. The models suggest that the advancing ice margin had thicknesses of between 250 and 550 km.

The possibility that this readvance may have been responsible for the crustal subsidence seen in the archeological and historical records was proposed and considered qualitatively by Weidick [1993, 1996]. Here we convolve an advancing ice sheet with our viscoelastic Green’s functions to see if the response of the Earth could be large enough to explain the significant GPS subsidence at Kellyville.

We convolve the viscoelastic Green’s functions described in section 3 (though now keeping all spherical harmonics up to degree and order 500) with a model of an advancing ice margin. To construct this ice model, we use a \((2.5 \text{ km} \times 2.5 \text{ km})\) representation of the present-day Greenland land/ice distribution (obtained from J. Bamber (personal communication, 1999)). We obtain a starting model by deglaciating all ice grid points lying within a distance \( x \) km to the east of the present western ice margin, between latitudes 63° and 70°N. We take the remaining ice-covered points as the starting model at an initial time of \( T_0 \) years before present. We let the ice sheet margin in this latitude band advance linearly with time, so that it reaches its present position at a time \( T_1 \) before present. We assume this advancing ice sheet has ice thickness \( H \). We vary the parameters \( x, H, T_0, \) and \( T_1 \) as well as the viscoelastic parameters used to compute the Green’s functions (i.e., the lithospheric thickness and the upper and lower mantle viscosities).

We find that our predicted uplift rates at Kellyville scale linearly with \( H \) and approximately linearly with \( x \) and are reasonably independent of \( T_1 \) as long as \( T_1 \) is no larger than a few hundred years. Figure 7 shows the uplift rates as a function of time for four representative models. Both the elastic and viscoelastic responses are included. The elastic parameters are the same for each model and are described in Table 1. All models assume that the ice thickness \( H \) is 400 m and that the readvance ended 150 years ago (\( T_1 = 150 \) years). Models A, B, and D assume the readvance started 3000 years ago (\( T_0 = 3000 \) years), while model C assumes it began 4000 years ago. Model B uses our default values to describe the viscous parameters (see section 3). Models A, C, and D assume the upper mantle viscosity is \( 0.6 \times 10^{21} \text{ Pa} \)
Figure 7. The predicted Kellyville uplift rate as a function of time caused by the proposed readvance of the western ice sheet. The four models are described in the text.

s, and model D also assumes a lithospheric thickness of 80 km. Our default values are used for all other viscous parameters in models A, C, and D.

In all cases the predicted uplift rates increase with time, because more ice is continually being added as the margin advances. The rates begin to decrease during the last 150 years after the margin is assumed to have stabilized. Rates tend to be larger for a thinner lithosphere, a smaller upper mantle viscosity, and a more recent start to the readvance (assuming the total distance of readvance \( x \) is held fixed).

The uplift rates at time \( t = 0 \) are the rates at the present time and can be compared with the \(-7.8 \pm 3.6 \text{ mm/yr} \) discrepancy between our Kellyville GPS uplift rate and the predicted PGR rate due to changes in ice prior to 4000 years ago. Note that the predicted trend for each of the three models considered in Figure 7 lies within our observational limits. In fact, the present-day uplift rate from model A (\(-7.3 \text{ mm/yr} \)) is similar to our preferred \(-7.8 \text{ mm/yr} \) GPS rate residual. This does not imply that the GPS results support the viscoelastic profile used in model A because the predicted results also depend strongly on the total distance of the ice sheet readvance \( x \) and on the ice thickness \( H \), which are not well known. Furthermore, the GPS residual uplift rate depends on the PGR rate we removed from the original GPS trend, and that PGR rate is highly uncertain. However, the results do suggest that the anomalous GPS uplift rate at Kellyville could quite possibly be the result of the Earth’s response to a readvancing ice sheet.

6. Summary and Discussion

We installed permanent GPS receivers at two locations along the edge of the ice sheet in southern Greenland: one at Kellyville (installed in 1995), along the western ice sheet margin, and the other at Kulusuk (installed in 1996), along the eastern margin and across the ice from Kellyville. Our primary objective is to determine the secular rate of vertical motion at each location. Any significant secular vertical motion would most likely be due to a combination of the Earth’s elastic response to ongoing secular changes in nearby ice and the Earth’s viscoelastic response to past ice variability.

It is not possible to separate the effects of these two processes using vertical displacement measurements alone. Separation is possible by combining the GPS secular uplift rate with measurements of the secular rate of change in gravity at the same location. Consequently, we have visited each site with an absolute gravimeter for \(-1-2 \text{ weeks every summer, beginning in 1995 at Kellyville and 1996 at Kulusuk.} \)

Our GPS results at Kulusuk suggest a secular uplift rate of \(-2.1 \pm 1.5 \text{ mm/yr} \), which is not notably different from zero, but our GPS results at Kellyville indicate a significantly nonzero uplift rate of \(-5.8 \pm 1.0 \text{ mm/yr} \). The Kellyville result is consistent with archeological and historical evidence of widespread subsidence of this region over the last 3000 years [Weidick, 1993, 1996].

There have not yet been enough gravity occupations at either site to warrant the inclusion of the gravity data in this analysis. If the Kellyville uplift rate of \(-5.8 \pm 1.0 \text{ mm/yr} \) does, indeed, mostly represent the Earth’s viscoelastic response to past loading, then the corresponding trend in gravity can be estimated by dividing that uplift rate by \(-6.5 \text{ mm/µGal (see (1)), resulting in an expected trend of } 0.9 \pm 0.2 \text{ µGal/yr.} \)

This trend is too small to be resolved by the Kellyville gravity measurements shown in Figure 3, ow-
ing to their large uncertainties. The formal errors on
the Kellyville gravity estimates lead to a ±1.2 μGal/yr
uncertainty in the trend. When this trend is inserted
into (1), the resulting uncertainty in Δ is ±7.2 mm/yr,
which is larger in magnitude than the −5.8 mm/yr rate
for υ inferred from the GPS data. If there continue to be
yearly gravimeter occupations at Kellyville, the formal
uncertainty in the trend should decrease inversely as the
number of those occupations to the 3/2 power (the effects
of the trend grow linearly with the data span, and the
formal error decreases inversely with the square
root of the number of occupations). Thus, in order for
the total uncertainty in Δ to equal, say, ±2.0 mm/yr
(≈1/3 of the GPS uplift rate at Kellyville), there will
need to be another 4–5 yearly gravity occupations of
Kellyville.

In the meantime, the GPS results can only be used
to constrain the sum of the elastic and the viscoelastic response to past
changes in ice. The viscoelastic response to changes in ice that are likely to have occurred prior to 4000 years
ago is probably causing Kellyville to uplift, rather than to subside. Our best prediction of this uplift rate, based
on convolving the viscoelastic Green’s functions with ice
deglaciation models of Tushingham and Peltier [1991],
Huybrechts [1994], and Le Meur and Huybrechts [1998],
is 2.0 ± 3.5 mm/yr. Removing this rate from the GPS
uplift rate gives a residual uplift rate at Kellyville of
−7.8 ± 3.6 mm/yr.

This is a far larger rate than is likely to be caused by
the Earth’s elastic response to present-day changes in ice. Instead, the most plausible explanation is that the
uplift rate reflects the Earth’s viscoelastic response to
changes in ice that occurred during the last 4000 years.

Evidence from the geological dating of moraines along
the west Greenland coast [van Tatenhove et al., 1995]
and the output from one of the two dynamical simu-
lations of Greenland ice considered here [Huybrechts,
1994] suggest that the ice along the southern third of
the western ice sheet margin began advancing ~3000-
4000 years ago and continued advancing for ~50 km,
reaching its present position a few hundred years ago.
Weidick [1993, 1996] noted that this advance, which
followed several thousand years of ice sheet recession,
could well have caused significant elastic and viscoelastic subsidence throughout this region.

By convolving our elastic and viscoelastic Green’s
functions with plausible models of this ice sheet read-
vance, we predict uplift rates at Kellyville that are
within the −7.8 ± 3.6 mm/yr bound for the residual
GPS uplift rate. We tentatively propose that an ice
sheet readvance did exist and that it is responsible for
the large negative uplift rate at Kellyville. In that case
the anomalous crustal subsidence is likely to be limited
to the southern third of the western ice sheet margin
and probably to within 200–300 km of either side of the
present margin. This subsidence is not likely to extend
into other regions under the ice sheet.

For example, Figure 8 shows the predicted, present-
day uplift rate across southeastern Greenland, com-
puted for model A, defined above in our description of
Figure 7. Note that the ice readvance would cause no
significant vertical motion at Kulusuk, where our GPS
trend is not significantly different from zero. Further-
more, we are not aware of any evidence that suggests
the eastern margin of the ice sheet in the vicinity of
Kulusuk might have undergone the sort of readvance
we are considering at the western margin.

Suppose the effects of viscoelastic crustal uplift are
removed from present or future altimeter observations
in order to improve the information those observations
provide on ongoing changes in ice thickness. Suppose
these viscoelastic estimates do not include the effects
of changes in ice subsequent to 4000 years ago. The results
shown in Figure 8 suggest that the resulting altimeter
estimates of the secular ice thickness change could then
include errors of 5–10 mm/yr along the southern third
of the western ice sheet margin, though those errors
would probably not persist more than 200–300 km into
the interior of the ice sheet.

Figure 8. The predicted present-day uplift rate (in
mm/yr) caused by the proposed readvance of the west-
ern Greenland ice margin, computed using model A.
The locations of Kellyville and Kulusuk are shown with
plus signs. The solid strip running primarily from north
to south near Kellyville is the total region assumed cov-
ered by this ice readvance.
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